Physics of Materials

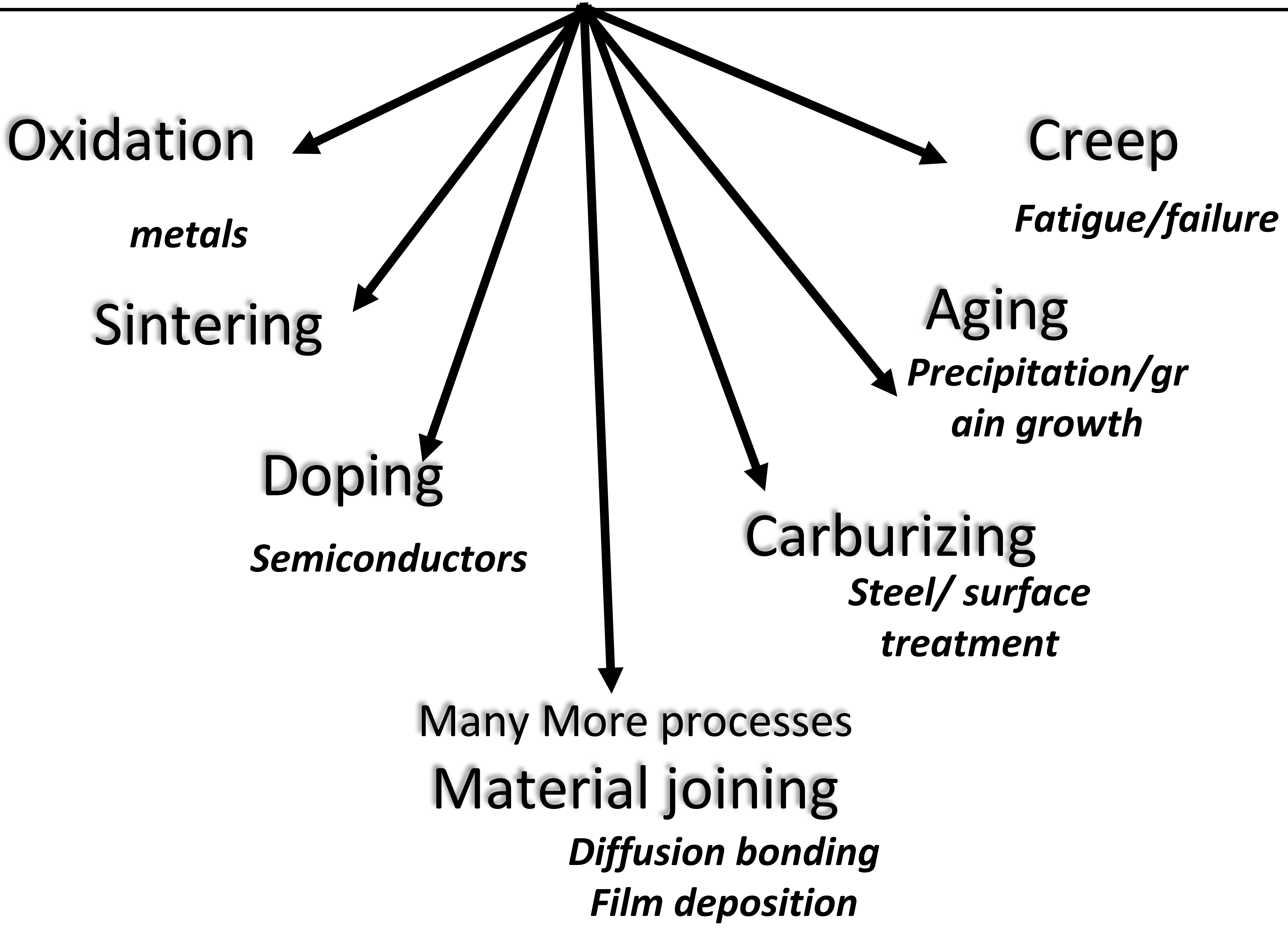
Chapter 5: Diffusion

Dr. Thomas LaGrange

Masters Course PHYS-307

Fall 2025

Roles of Diffusion



Controlling diffusion is essential for fabricating Novel Spintronic and Quantum nanomaterials

PHYSICAL REVIEW B **107**, 094408 (2023)

Unraveling the sign reversal of the anomalous Hall effect in ferromagnet/heavy-metal ultrathin films

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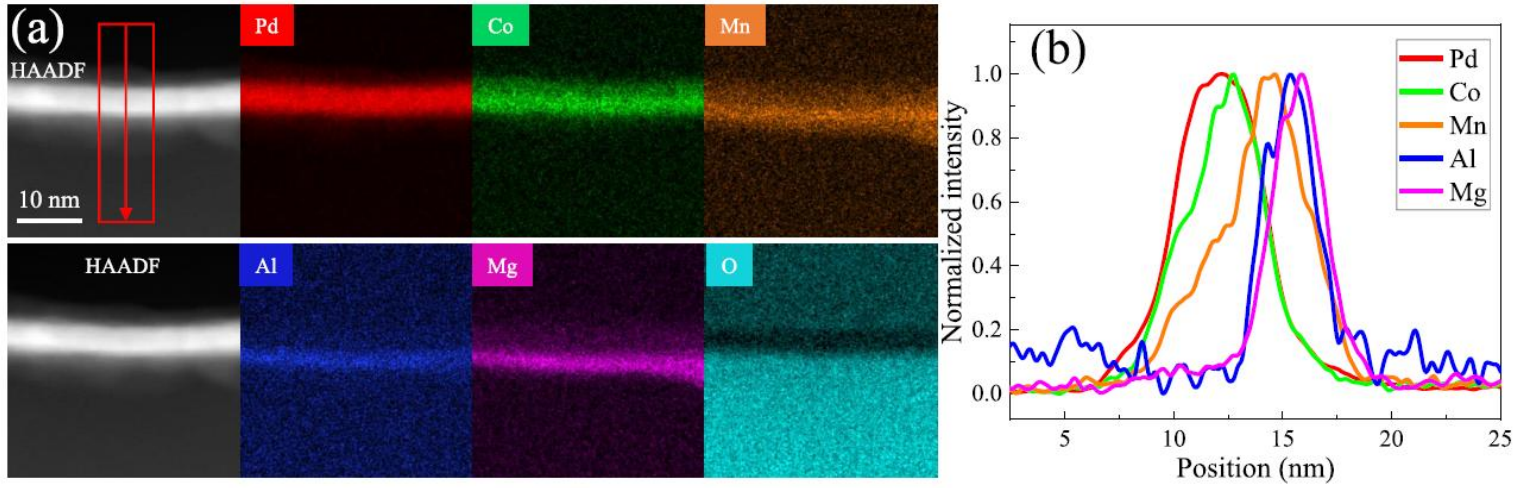
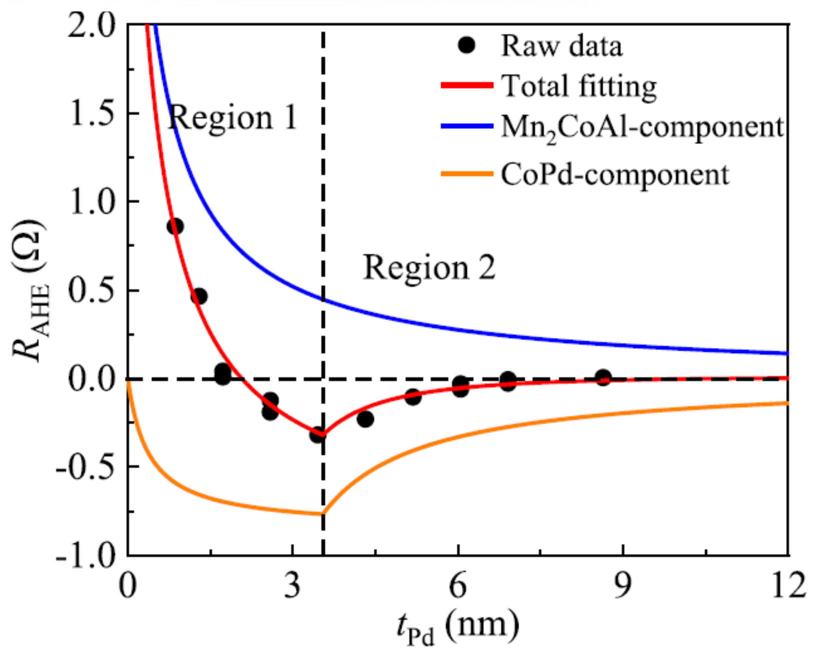
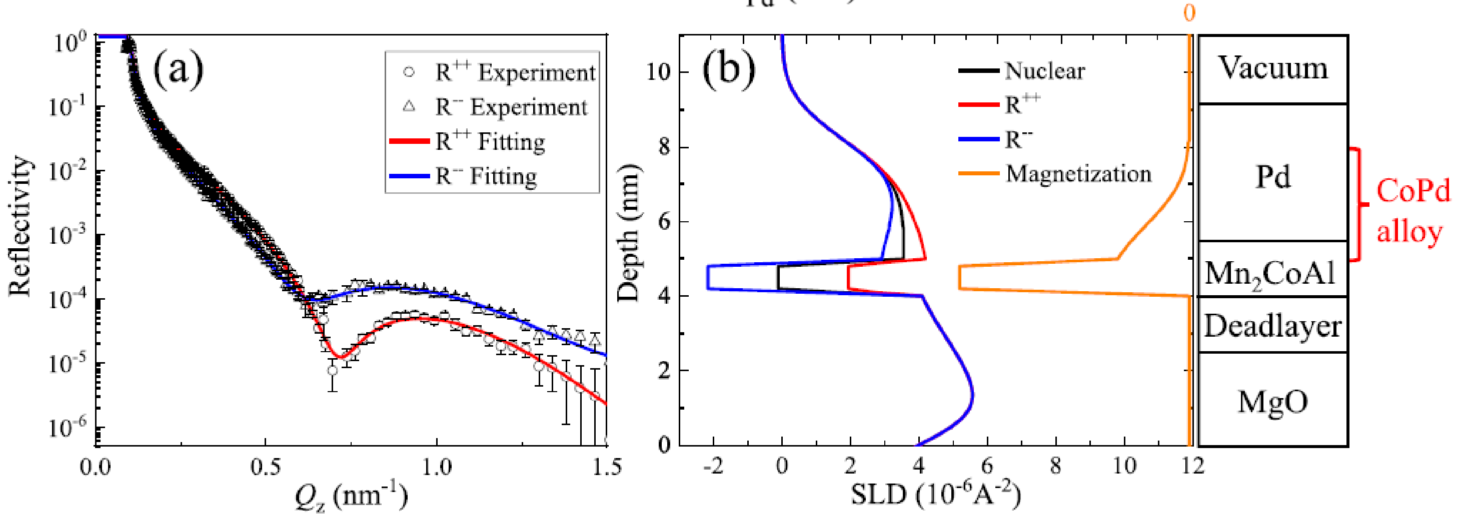
(Received 12 January 2023; accepted 28 February 2023; published 10 March 2023)

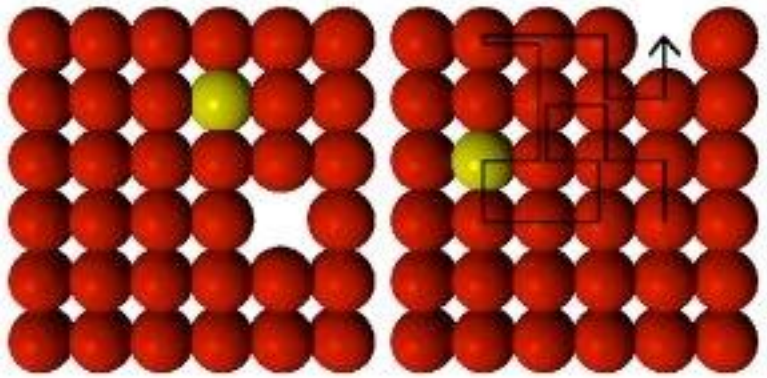
The sign reversal in the anomalous Hall effect (AHE) that occurs for material offers great prospects for AHE-based spintronic devices design. However, the mechanisms are still controversial in ultrathin ferromagnetic/heavy metal thin film systems due to the complicatedly interfacial effects. Here, we investigate the AHE sign reversal in ultrathin ferromagnetic Mn_2CoAl/Pd films, a system which has shown unusual AHE, significant spin-orbit coupling, and magnetic texturing. Element-sensitive cross-sectional STEM imaging and the depth-resolved magnetization profile from polarized neutron reflectometry identifies the presence of a second ferromagnetic layer from intermixed Co-Pd. To quantitatively explain the sign reversal of the AHE, we build a model based on two contributions, ferromagnetic Mn_2CoAl and the intermixed CoPd layer. We also clarify that contributions to the AHE from magnetic proximity and spin Hall effect are negligible. Our work demonstrates that interfacial alloying can be a critical factor and provides insightful methods to determine the origins of the AHE in ferromagnet/heavy-metal thin film systems.

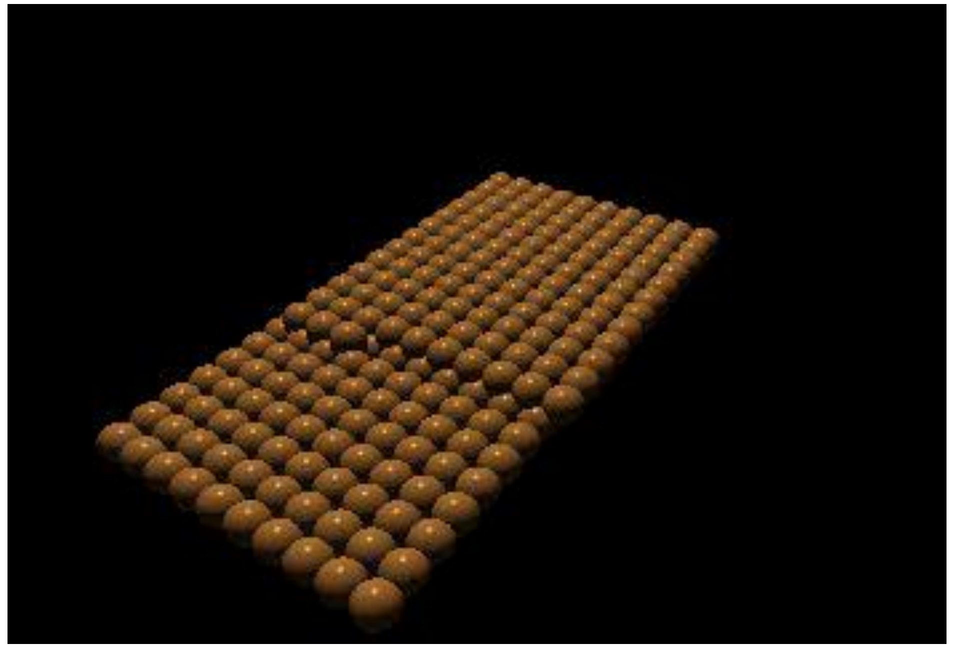


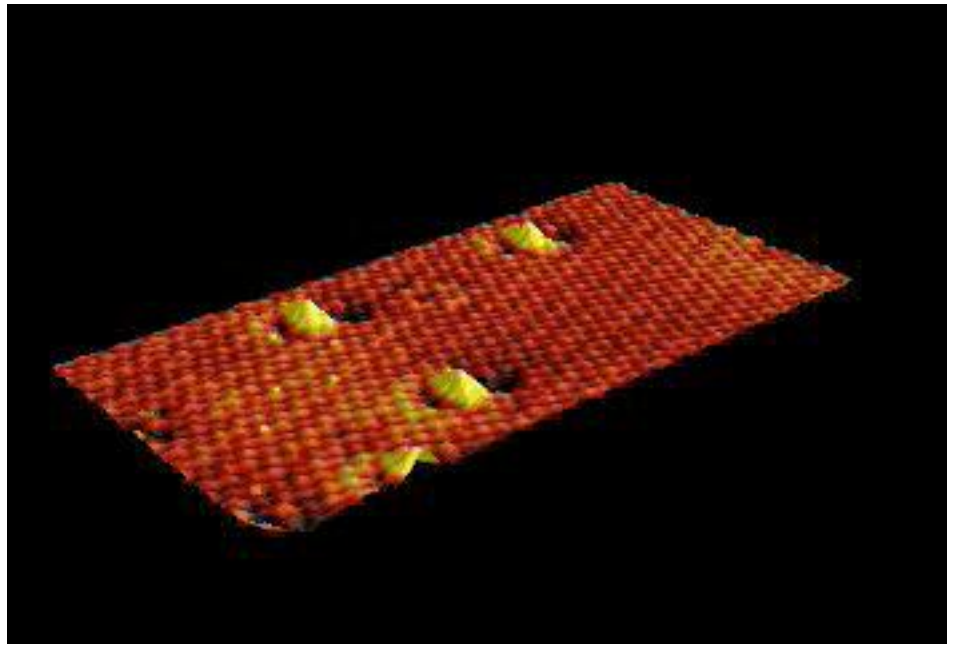




The puzzle







vacancies

Fundamental equations of diffusion

Fick's laws

$$\vec{J}_A = -D_A \overrightarrow{grad} C_A$$ 1st law (steady state)

$$\frac{\partial C_A}{\partial t} = -div \vec{J}_A$$ 2nd law(non-steady state)

$$\downarrow$$

$$\frac{\partial C_A}{\partial t} = D_A \Delta C_A$$

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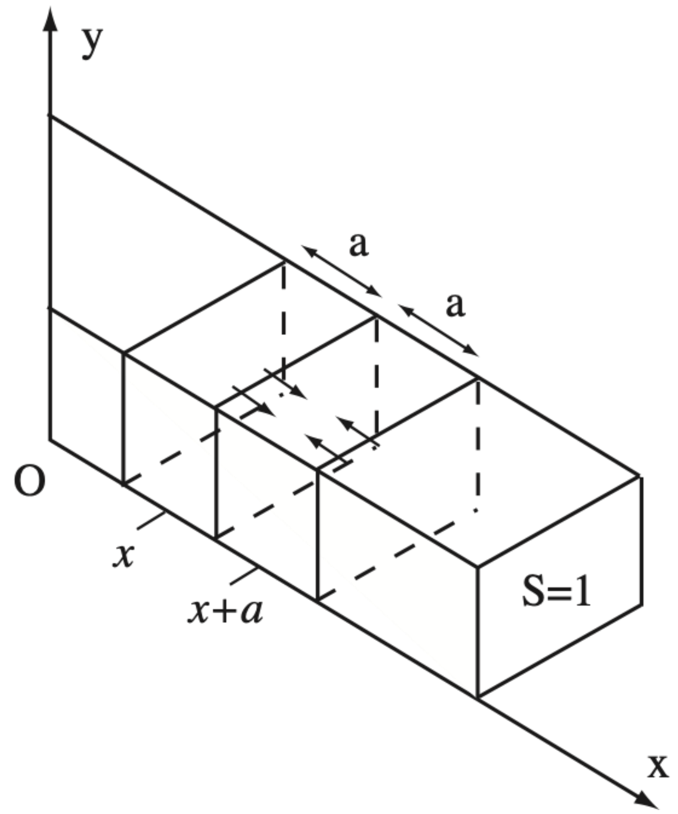
Demonstration of the 1st law in a one-dimensional system

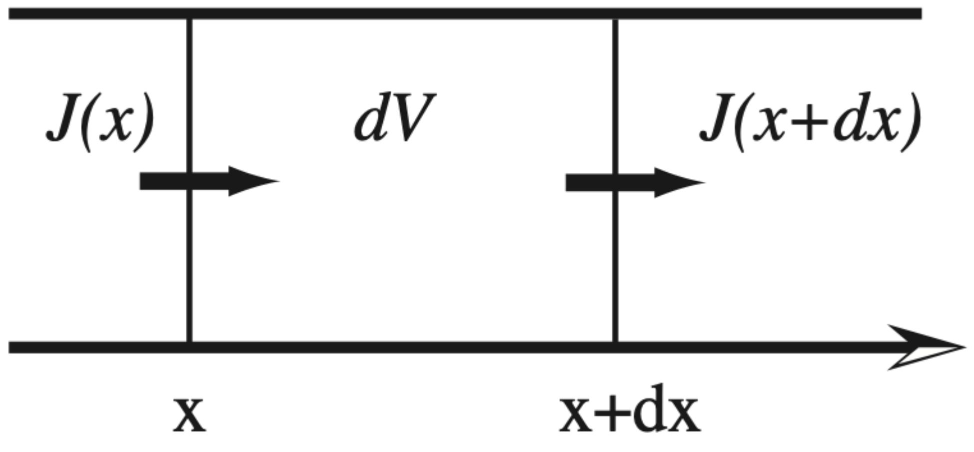


$$J_A = \frac{1}{2}\Gamma C_A(x)a - \frac{1}{2}\Gamma C_A(x+a)a$$

$$J_A = \frac{1}{2}\Gamma a\left[C_A(x) - C_A(x+a)\right] = -\frac{1}{2}\Gamma a^2 \frac{\partial C_A}{\partial x}$$

thus $D_A = \frac{1}{2}\Gamma a^2$

Demonstration of the 2nd law in a one-dimensional system



$$dV = Sdx$$

$$\frac{dn}{dt} = \left(J(x) - J(x+dx)\right)S = -S\frac{\partial J}{\partial x}dx = -\frac{\partial J}{\partial x}dV$$

$$\frac{1}{dV}\frac{dn}{dt} = \frac{dC}{dt} = -\frac{dJ}{dx}$$

in 3D: $\frac{dC}{dt} = -div\vec{J}$

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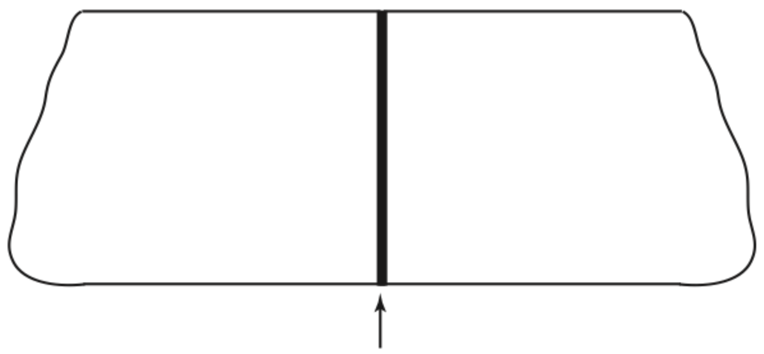
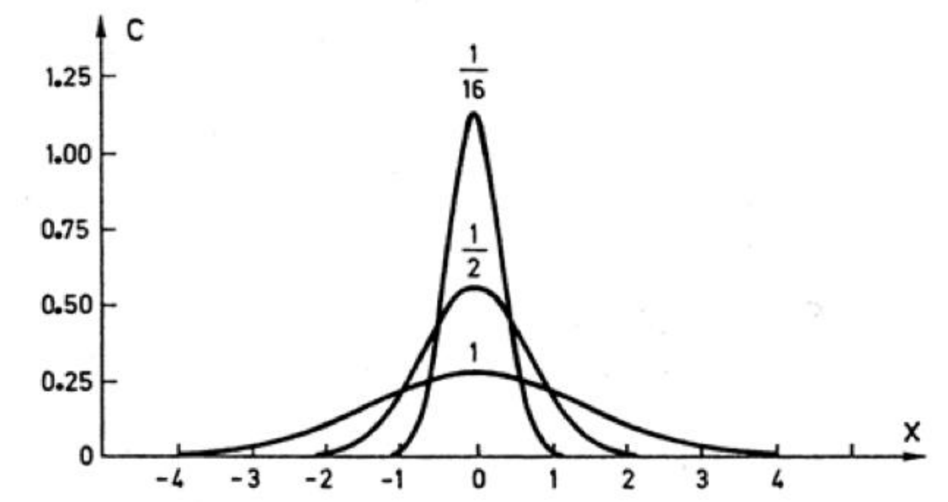
Solutions of the Fick's second law: C(x,t)

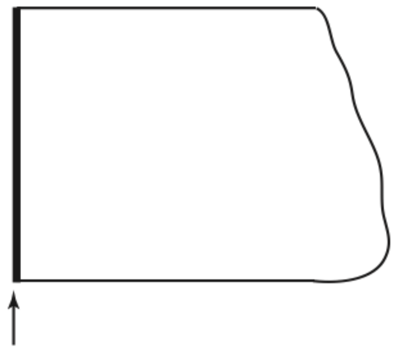
Thin layer of an element B fully soluble in A





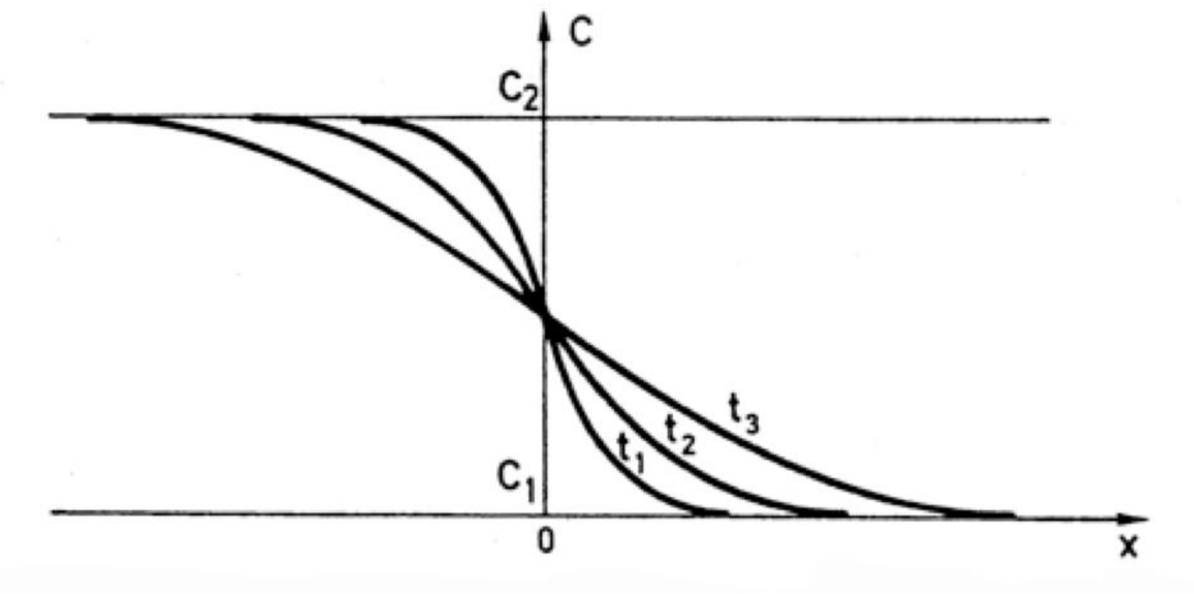
Surface layer: same solution

$$C(x,t) = \frac{M}{2\sqrt{\pi D t}} e^{-\frac{x^2}{4Dt}}$$



Solutions of the 2nd Fick equation: C(x,t)

Interdiffusion



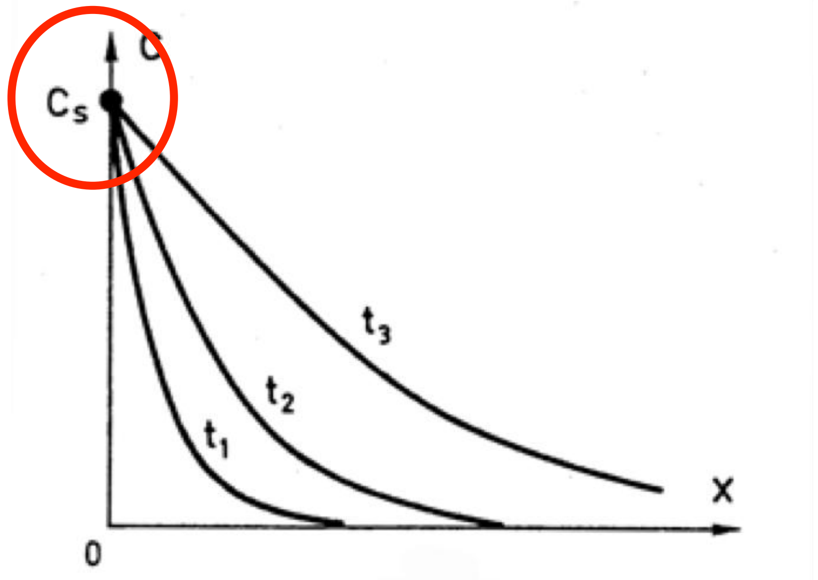
At $t=0$, $C=C_0$ for $-\infty < x < 0$

$C=0$ for $0 < x < \infty$

$$C(x,t)=\frac{C_0}{2}\left(1-\theta\left(\frac{x}{2\sqrt{Dt}}\right)\right)$$

$$\theta(y)=\frac{2}{\sqrt{\pi}}\int_0^y e^{-z^2}\,dz$$ is the error function

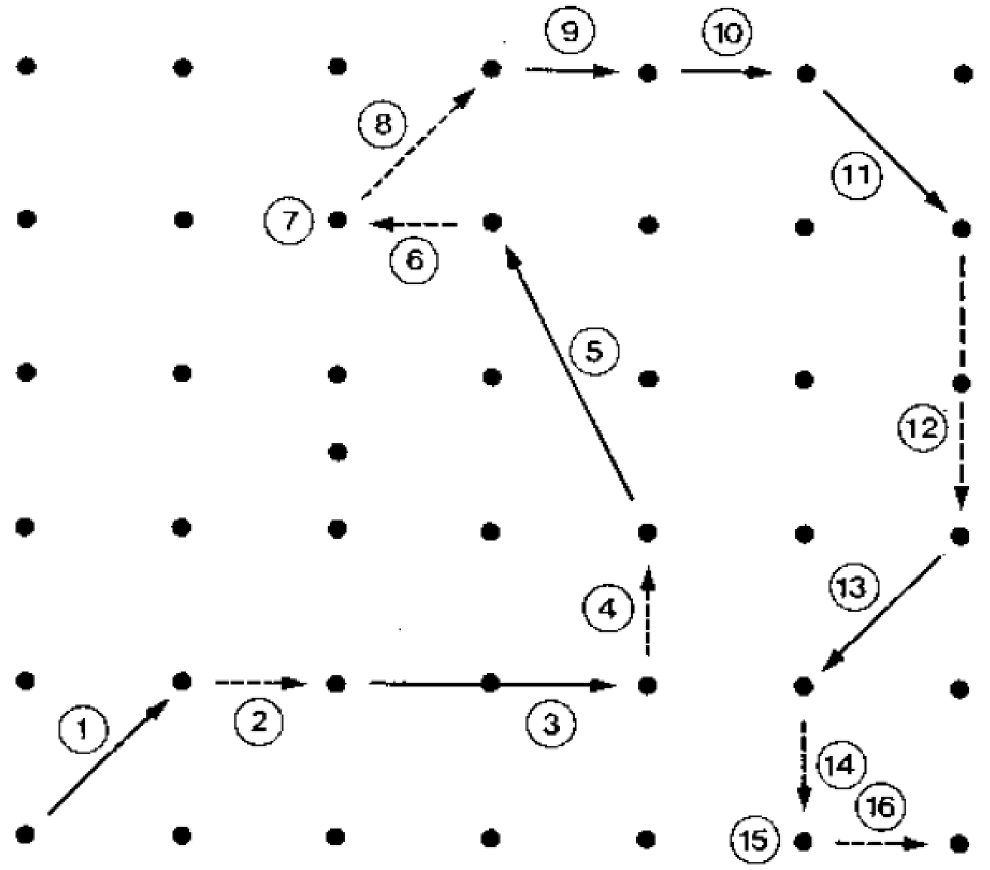
Diffusion from a surface at constant concentration



$$\frac{C(x,t)-C_0}{C_s-C_0}=\left(1-\theta\left(\frac{x}{2\sqrt{Dt}}\right)\right)$$

Diffusion coefficient and random walk

migration along vectors $\overrightarrow{\delta l}$ corresponding to dense directions



$$\vec{L} = \sum_i \overrightarrow{\delta l}_i$$

$$\langle \vec{L} \rangle = 0$$

$$\langle \vec{L}^2 \rangle \neq 0$$

$$\langle \vec{L}^2 \rangle = \left\langle \sum_i \overrightarrow{\delta l}_i^2 \right\rangle + \left\langle \sum_{ij} \overrightarrow{\delta l}_i \overrightarrow{\delta l}_j \right\rangle$$

$$\langle \vec{L}^2 \rangle = \left\langle \sum_{i=1}^{n} \overrightarrow{\delta l}_i^2 \right\rangle = n\overrightarrow{\delta l}^2 = \Gamma t \overrightarrow{\delta l}^2$$

$$\vec{L}^2 = x^2 + y^2 + z^2 \Rightarrow \langle x^2 \rangle = \frac{1}{3} \langle \vec{L}^2 \rangle = \frac{1}{3} \Gamma t \delta l^2$$

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Particle flux though a surface S

$$\Phi = Jt = \frac{1}{2}C(x)\Delta - \frac{1}{2}C_A(x+\Delta)\Delta$$

$$J = -\frac{1}{2}\frac{\Delta^2}{t}\frac{\partial C(x)}{\partial x} = -\frac{1}{2}\frac{\langle x^2\rangle}{t}\frac{\partial C(x)}{\partial x} \Rightarrow \boxed{D = \frac{1}{2}\frac{\langle x^2\rangle}{t}} \text{ or } \langle x^2\rangle = 2Dt$$

$$\boxed{D = \frac{1}{6}\Gamma\delta l^2}$$ simple cubic

coordination number

$$\boxed{D = \frac{1}{8}\Gamma\delta l^2}$$ body-centered cubic

$$\boxed{D = \frac{1}{12}\Gamma\delta l^2}$$ face-centered cubic

Dependence with respect to temperature

$D=D(T)$ since $\Gamma = \Gamma(T)$

$$\Gamma = \nu_D P$$

ν_D is the Debye vibration frequency ~ 10^{13} Hz
Also known as the phonon frequency

$P \sim e^{-\frac{\Delta G}{kT}}$ is the probability that the atom has a sufficient energy to jump from one site to another

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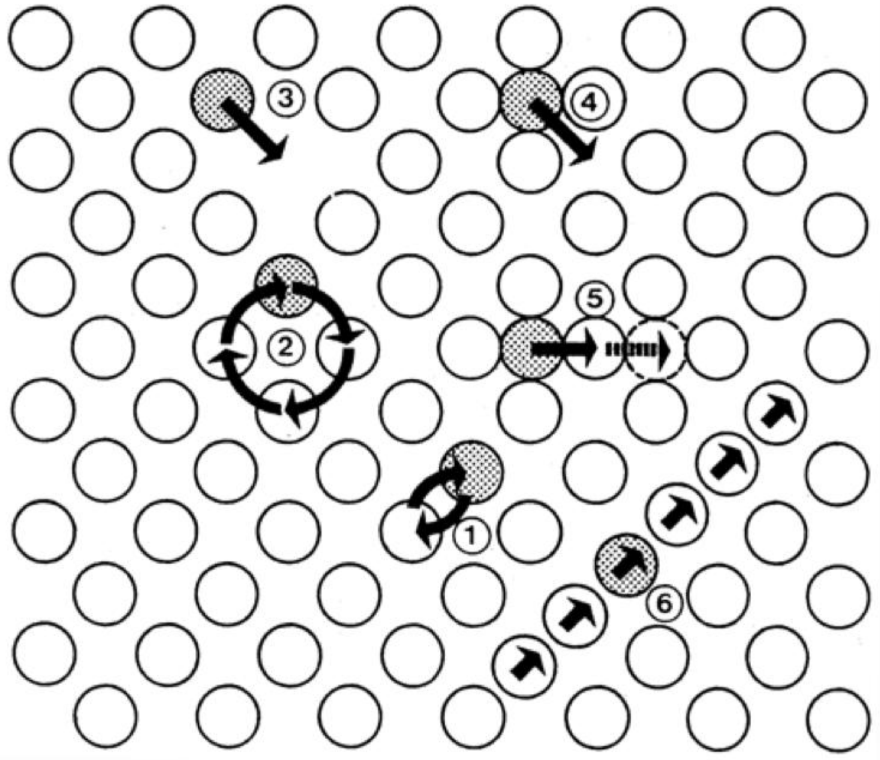
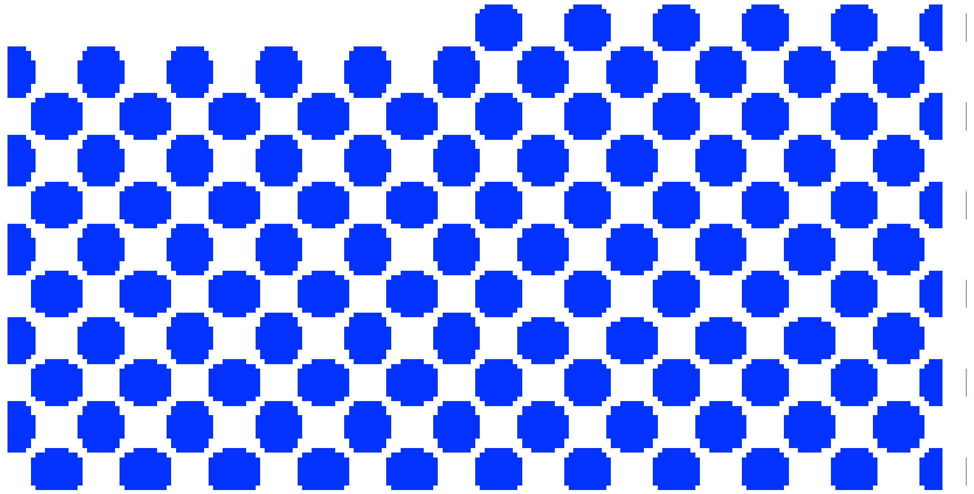
Self-diffusion





vacancy mechanism

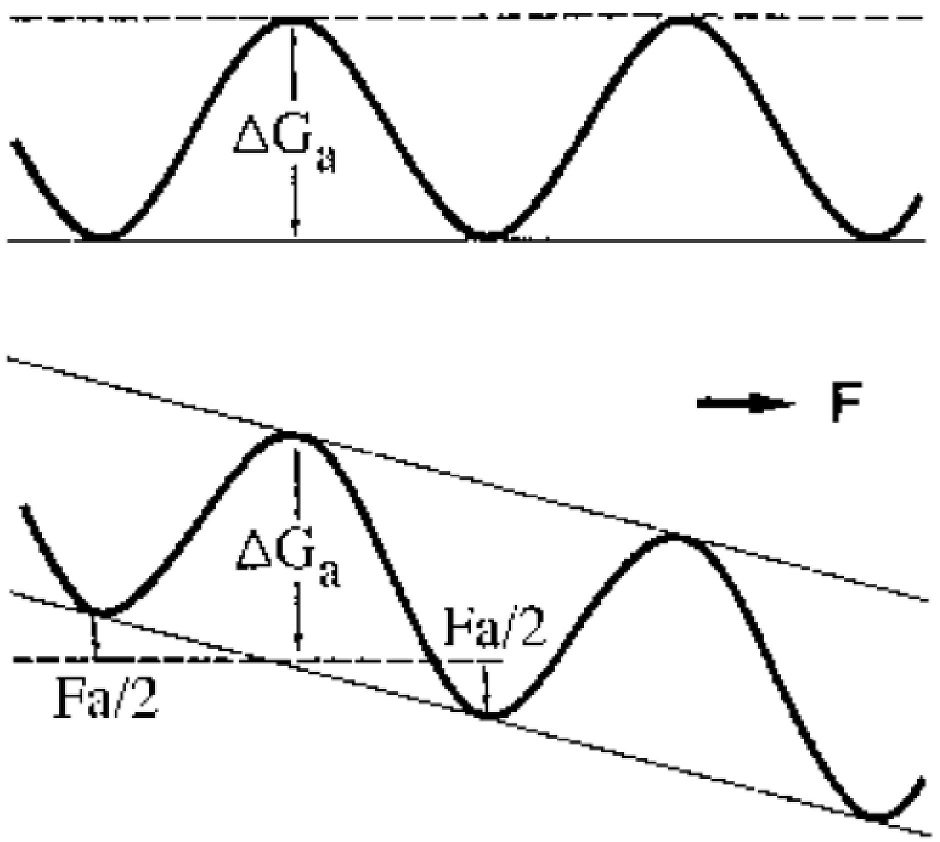
$$\Gamma = z\nu_D C_V \exp\left(-\Delta G_V^m / kT\right) = z\nu_D \exp\left(-\left(\Delta G_V^m + \Delta G_V^F\right)/kT\right)$$

$$D = \frac{1}{6} a^2 z\nu_D \exp\left(-\left(\Delta G_V^m + \Delta G_V^F\right)/kT\right)$$

$$D = \frac{1}{6} a^2 z\nu_D \exp\left(\left(\Delta S_V^m + \Delta S_V^F\right)/k\right) \exp\left(-\left(\Delta H_V^m + \Delta H_V^F\right)/kT\right) = D_0 \exp\left(-Q_{SD}/kT\right)$$

$$D_0 = \frac{1}{6} a^2 z\nu_D \exp\left(\left(\Delta S_V^m + \Delta S_V^F\right)/k\right)$$

Force applied to a diffusing particle: Einstein equation



$$v = \frac{1}{2} a \left(\Gamma^{+} - \Gamma^{-} \right)$$

$$\Gamma^{+} = \nu_0 e^{-\frac{\Delta G - \frac{a}{2}F}{kT}} \qquad \Gamma^{-} = \nu_0 e^{-\frac{\Delta G + \frac{a}{2}F}{kT}}$$

$$\overline{v} = \frac{1}{2} a \nu_0 e^{-\frac{\Delta G}{kT}} \left(e^{\frac{Fa}{2kT}} - e^{-\frac{Fa}{2kT}} \right) \approx \frac{1}{2} a^2 \nu_0 e^{-\frac{\Delta G}{kT}} \frac{F}{kT}$$

$$D_A = \frac{1}{2} a^2 \nu_0 e^{-\frac{\Delta G}{kT}}$$ and $$\boxed{v = \frac{D_A F}{kT}}$$

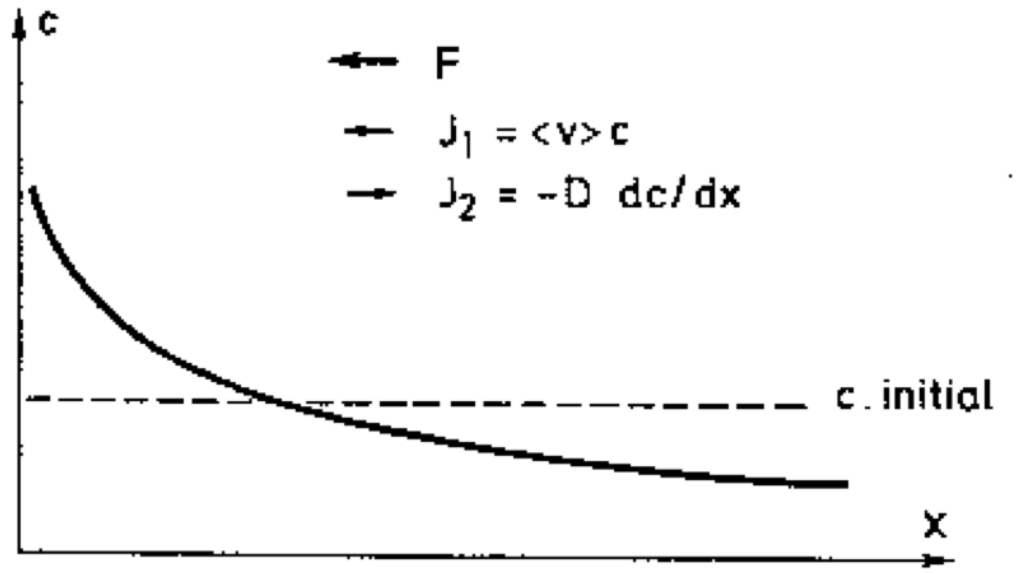
Einstein equation

Demonstration in the case where the force derives from a potential

Particle flux $J_1 = \langle v \rangle C$

Concentration gradient $J_2 = -D\dfrac{dC}{dx}$

$$J = 0 \Rightarrow D\frac{dC}{dx} = \langle v \rangle C$$



If F derives from a potential $F = -\dfrac{d\Phi}{dx}$

We suppose $C(x) = C_0 \exp(-\Phi(x)/kT)$

$$\frac{dC}{dx} = -\frac{C}{kT}\frac{d\Phi}{dx} = \frac{CF}{kT} \Rightarrow \langle v \rangle = \frac{FD}{kT}$$

Darken equation

For instance

$$\vec{F} = -\overrightarrow{grad}\mu_A$$

μ_A is the chemical potential

$$\mu_A = kT\ln C_A \Rightarrow F = -\frac{kT}{C_A}\frac{dC_A}{dx}$$

$$\vec{J}_A = C_A\vec{v}_A = -\frac{C_A D_A}{kT}\overrightarrow{grad}\mu_A = -\frac{C_A D_A}{kT}\frac{\partial\mu_A}{\partial C_A}\overrightarrow{grad}C_A = -D_A^* \cdot \overrightarrow{grad}C_A$$

$$D_A^* = \frac{C_A D_A}{kT}\frac{\partial\mu_A}{\partial C_A}$$

$$\frac{\partial C_A}{\partial t} = div\left(D_A^* \cdot \overrightarrow{grad}C_A\right)$$

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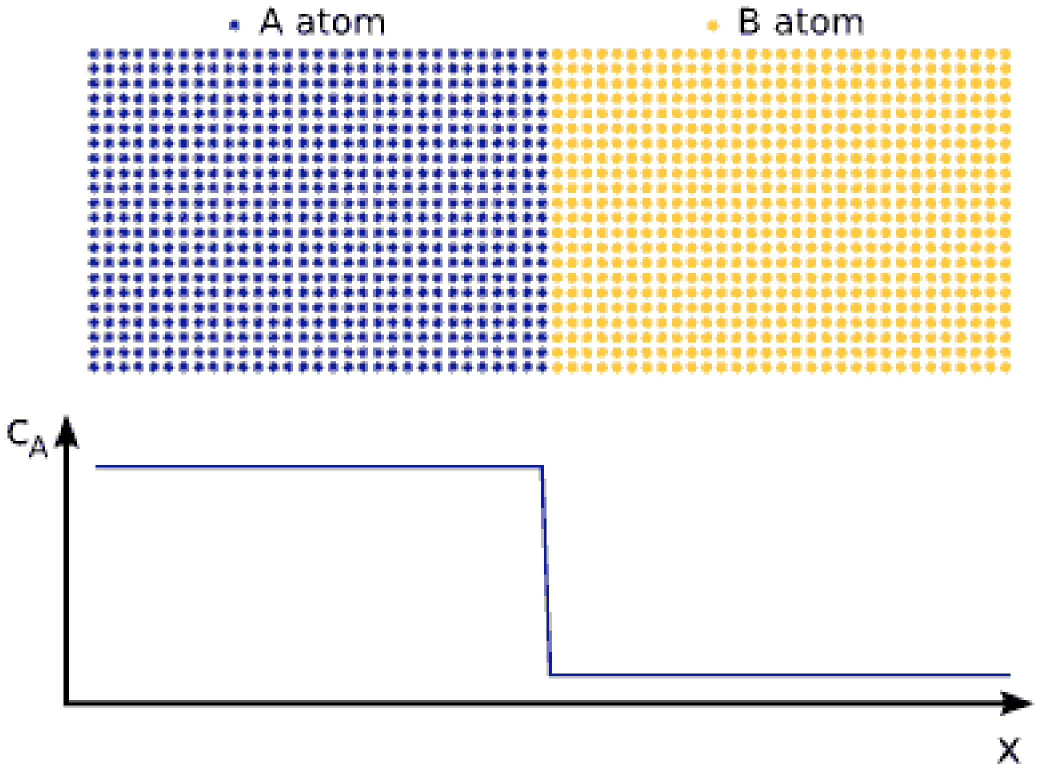
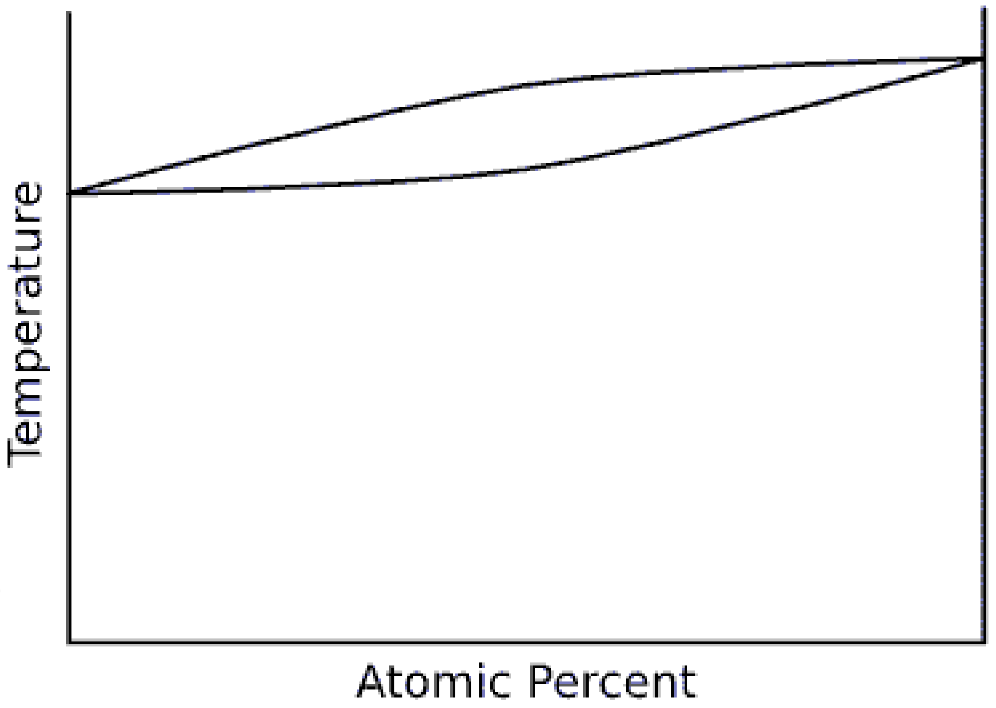
Interdiffusion : Boltzmann-Matano method



∂C



We must solve the one dimensional Fick equation

$$\frac{\partial C_A}{\partial t} = \frac{\partial}{\partial x}\left(D_A^{*} \cdot \frac{\partial C_A}{\partial x} \right)$$

Reminder on scaling laws

$$\ddot{\vec{x}}_1(t) = -\frac{GM}{|\vec{x}_1(t)|^3}\vec{x}_1(t) \qquad t' = \tau t \quad \vec{x}_1^{'} = \lambda \vec{x}_1$$

$$\ddot{\vec{x}}'_1(t) = -\frac{G'M\,\vec{x}'_1(t)}{|\vec{x}'_1|^3} \qquad \vec{x}_1^{'}(t=0) = \lambda \vec{x}_1(t=0)$$

$$\ddot{\vec{x}}_1(t) = \frac{\tau^2}{\lambda^3} G' \frac{M\,\vec{x}_1(t)}{|\vec{x}_1|^3} \Rightarrow \frac{\tau^2}{\lambda^3} G' = G$$

Consider another planet (2) characterized by:

$$\ddot{\vec{x}}_2(t) = -\frac{G'M\vec{x}_2(t)}{|\vec{x}_2(t)|^3} \qquad \vec{x}_2(t=0) = \lambda \vec{x}_1(t=0)$$

If $G = G' \Rightarrow \frac{\tau^2}{\lambda^3} = 1$ $\quad \vec{x}_1^{'}(t')$ and $\vec{x}_2(t)$ same trajectories

$$\left(\frac{t'}{t}\right)^2 = \left(\frac{x'_1}{x_1}\right)^3 = \left(\frac{x_2}{x_1}\right)^3$$ Kepler's 3rd law

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Scale changes in the diffusion equation

$$\frac{\partial}{\partial t}c_1(x,t)=\frac{\partial}{\partial x}\left(D\frac{\partial}{\partial x}c_1(x,t)\right) \qquad c_1(t=0)=c(x,0)$$

$$\frac{\partial}{\partial t'}c'_1(x',t')=\frac{\partial}{\partial x'}\left(D'\frac{\partial}{\partial x'}c'_1(x',t')\right) \qquad c'_1(t=0)=c(x',0)$$

$$\frac{\partial}{\partial t}c'_1(\lambda x,\tau t)=\frac{\partial}{\partial x}\left(\frac{\tau D'}{\lambda^2}\frac{\partial}{\partial x}c'_1(\lambda x,\tau t)\right) \qquad t'=\tau t \quad x_1'=\lambda x_1$$

New concentration profile $c_2(x,t)$

$$\frac{\partial}{\partial t_2}c_2(x_2,t_2)=\frac{\partial}{\partial x_2}\left(D\frac{\partial}{\partial x_2}c_2(x_2,t_2)\right) \qquad c_2(t=0)=c(\lambda x,0)$$

$$\frac{\tau D'}{\lambda^2}=D \qquad \frac{x}{\sqrt{t}}=\frac{x'}{\sqrt{t'}}$$

This can be deduced from the mean free path

$$2D'=\frac{\vec{x}'^2}{t'}=\frac{\lambda^2}{\tau}\frac{\vec{x}^2}{t}=2D\frac{\lambda^2}{\tau} \Rightarrow D'=\frac{D\lambda^2}{\tau}$$

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Solution of the diffusion equation

$$\frac{\partial C_A}{\partial t} = \frac{\partial}{\partial x}\left(D_A^* \cdot \frac{\partial C_A}{\partial x} \right)$$

Variable change $\eta = \frac{x}{\sqrt{t}}$

$$\frac{\partial}{\partial t} = \frac{\partial \eta}{\partial t}\frac{\partial}{\partial \eta} = -\frac{x}{2t^{3/2}}\frac{\partial}{\partial \eta} \qquad \frac{\partial}{\partial x} = \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta} = \frac{1}{\sqrt{t}}\frac{\partial}{\partial \eta}$$

We get

$$-\frac{\eta}{2}\frac{dC_A}{d\eta} = \frac{d}{d\eta}\left(D*_A \frac{dC_A}{d\eta} \right)$$

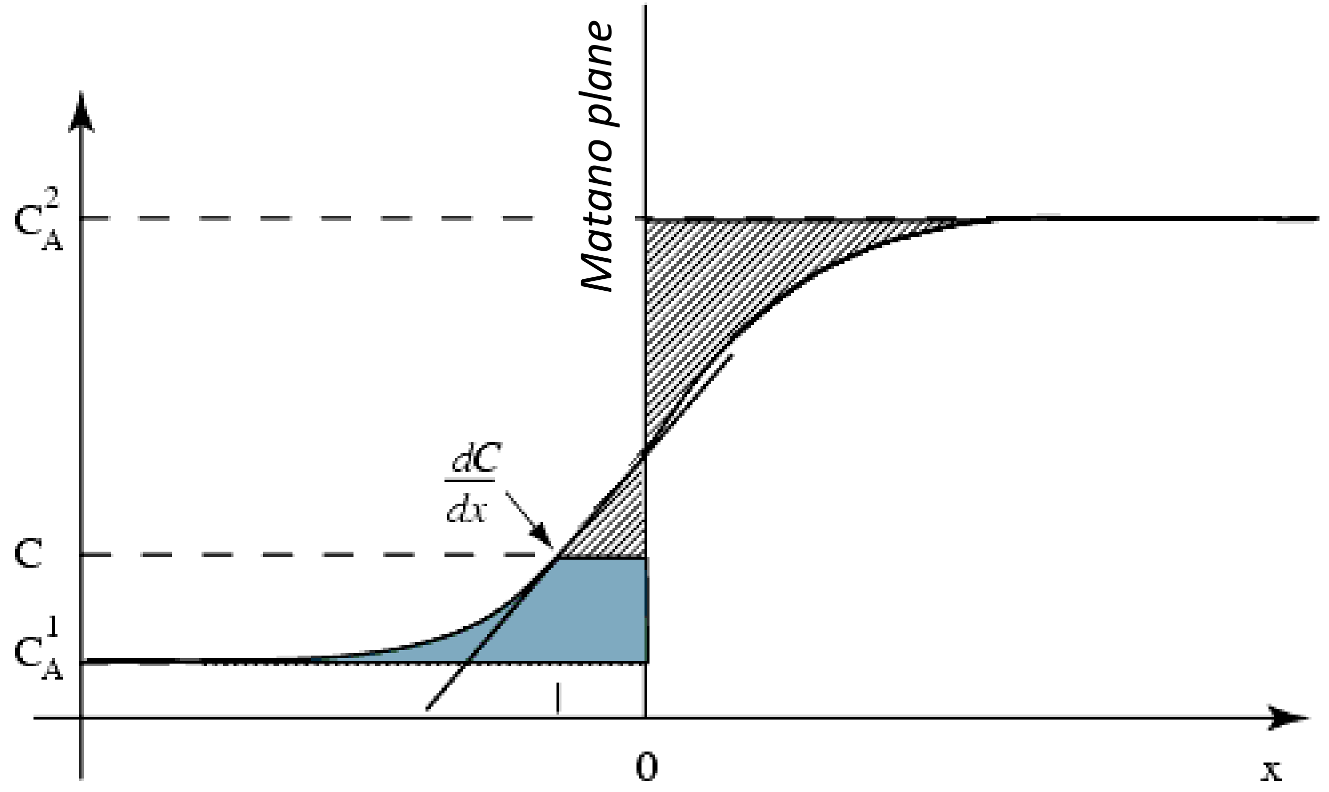
$$D*_A(C) = -\frac{1}{2}\frac{\int_{C_A}^{C} \eta \, dC}{\left.\frac{dC_A}{d\eta}\right|_C}$$

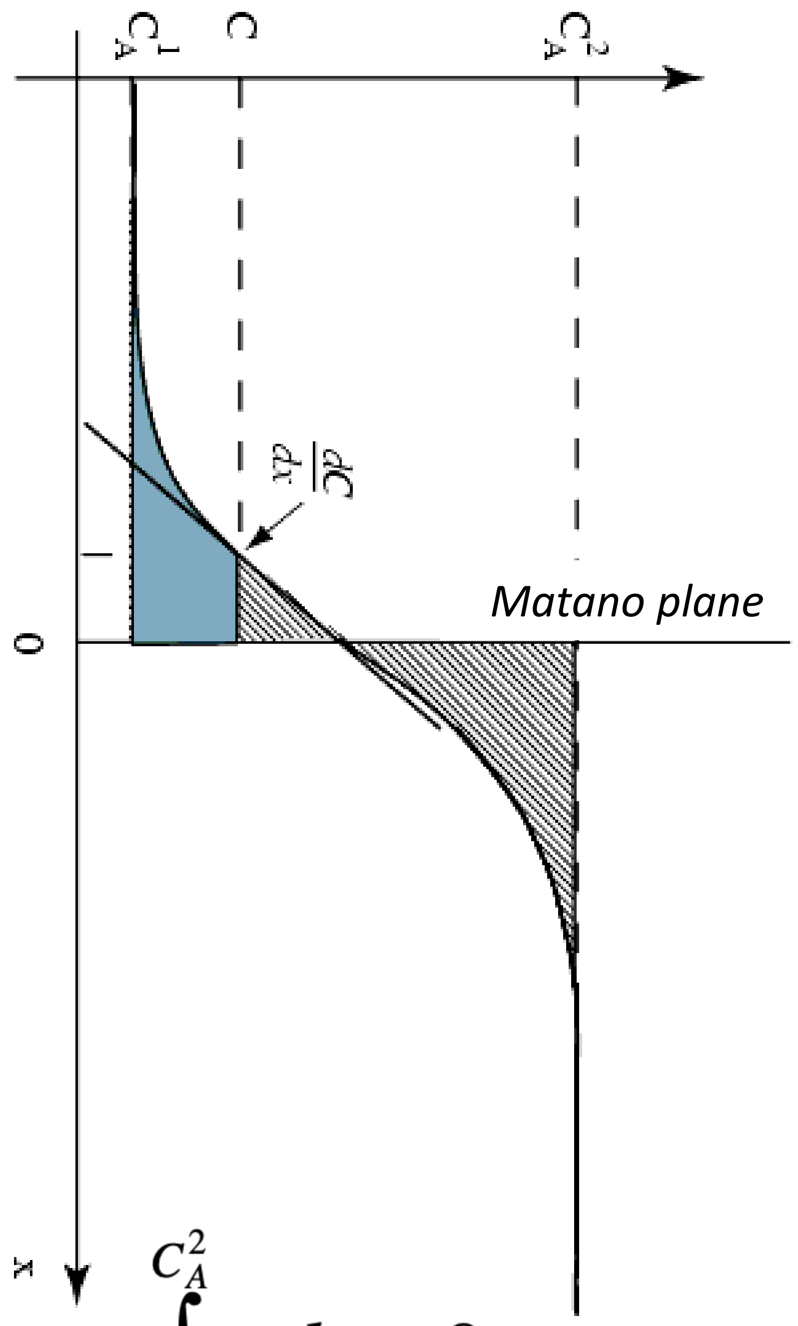
Solving the diffusion equation: Boltzmann-Matano method

In terms of variables x and t

$$D^*_A(C) = -\frac{1}{2}\frac{\int_{C_A^1}^{C} x\,dC}{\left.\frac{dC_A}{dx}\right|_C}$$

experimental method





$$\int_{C_A^1}^{C_A^2} x\,dc = 0$$

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Solving the diffusion equation: Heaviside step distribution

$$C_A(x,t=0)=\theta(x)$$

$$C_B(x,t=0)=\theta(-x)$$

$$\lim_{t\to 0}\left.\frac{\partial C_A}{\partial x}\right|_t=\delta(x)$$

$$D^*_A = constant$$

$$\frac{\partial C_A}{\partial t}=\frac{\partial}{\partial x}\left(D_A^*\cdot\frac{\partial C_A}{\partial x}\right)$$

$$-\frac{\eta}{2}\frac{dC_A}{d\eta}=D^*_A\frac{d^2}{d\eta^2}C_A$$

Consider a reduced variable $\eta=\frac{x}{\sqrt{t}}$

$$\frac{d}{d\eta}\left(C_A(\eta)\right)=ke^{-\eta^2/4D^*_A}$$

$$\left.\frac{\partial c}{\partial x}\right)_t=\frac{\partial c}{\partial \eta}\frac{\partial \eta}{\partial x}=\frac{1}{\sqrt{t}}\frac{\partial c}{\partial \eta}$$

$$\Rightarrow \quad \lim_{t\to 0}\left(\frac{k}{\sqrt{t}}e^{-\eta^2/4D^*_A}\right)=\delta(x)\ \Rightarrow\ k=\frac{1}{\sqrt{4D^*_A\pi}}$$

Solving the diffusion equation: Heaviside step distribution

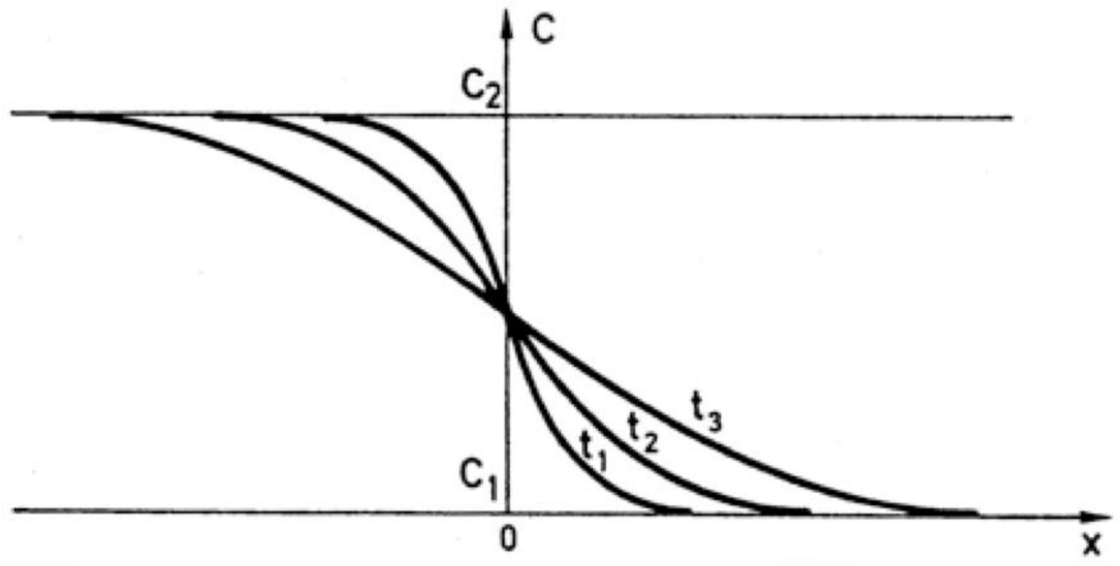
$$C_A(x,t=0)=\theta(x)$$

$$C_B(x,t=0)=\theta(-x)$$

$$\lim_{t\to 0}\left.\frac{\partial C_A}{\partial x}\right|_t=\delta(x)$$

$$D^*_A = constant$$

$$\frac{\partial C_A}{\partial t}=\frac{\partial}{\partial x}\left(D_A^*\cdot\frac{\partial C_A}{\partial x}\right)$$



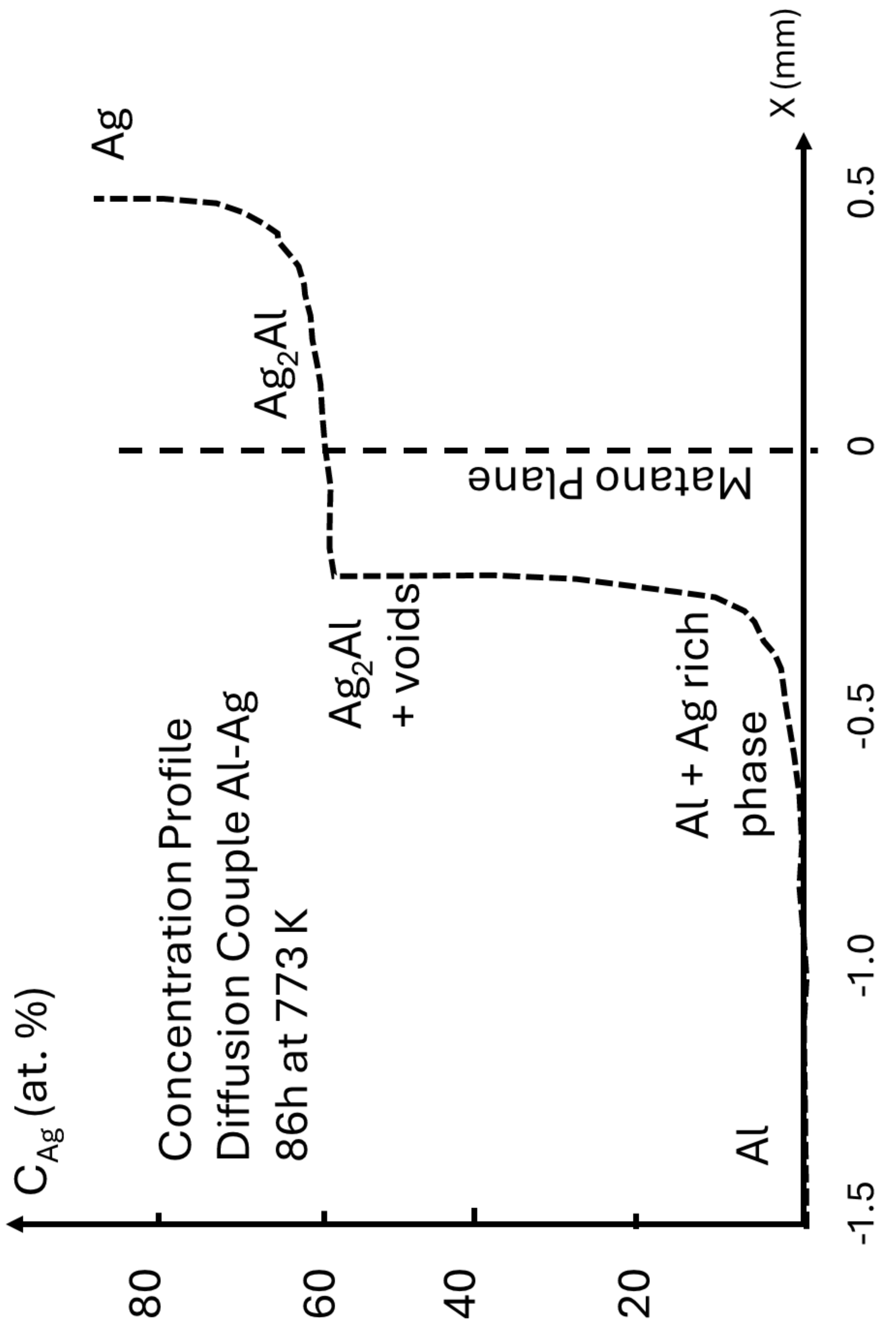
$$\frac{d}{d\eta}C_A=\frac{1}{\sqrt{4\pi D^*_A}}\exp\left[-\eta^2/4D^*_A\right]$$

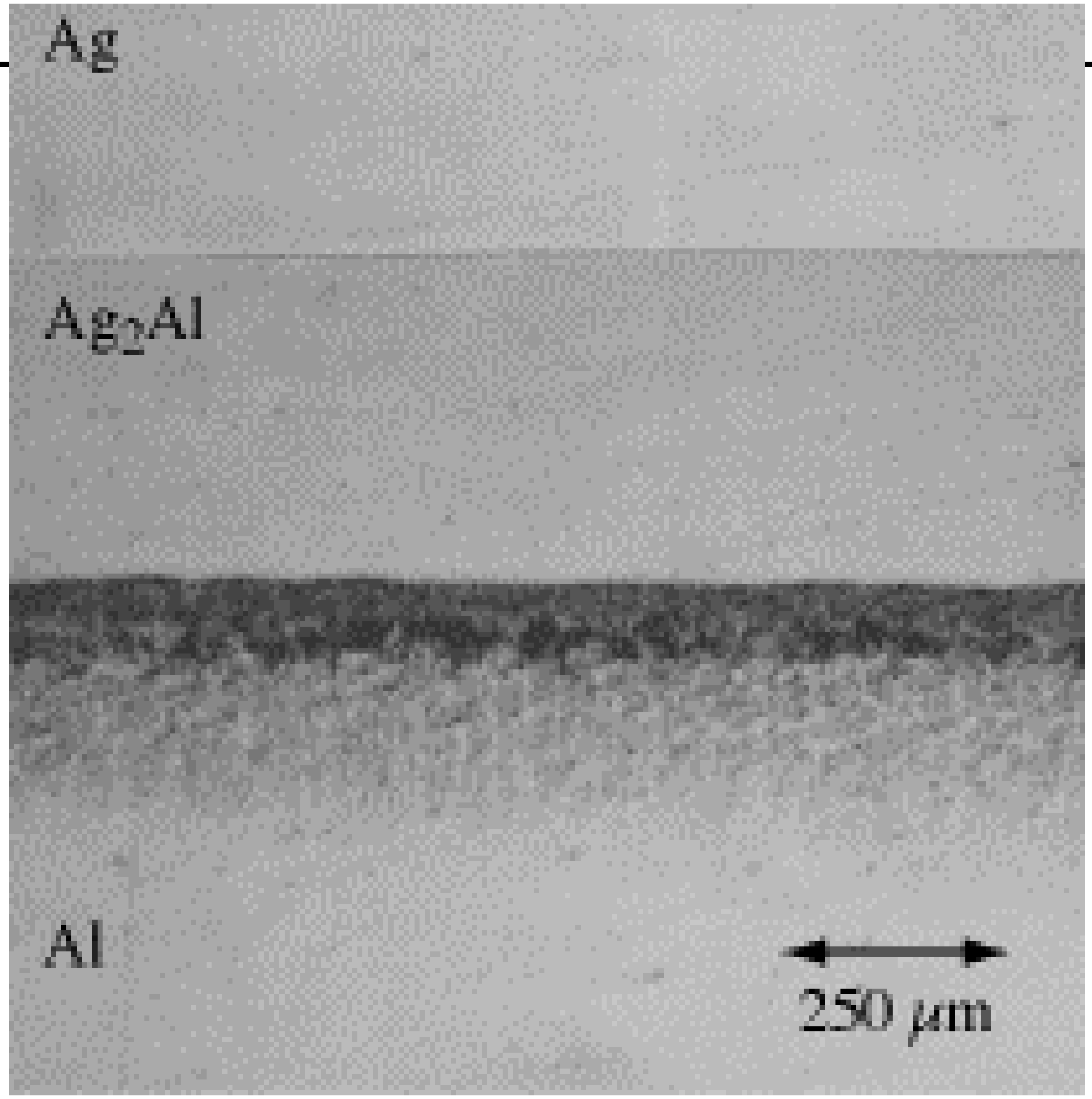
$$\frac{\partial C_A}{\partial x}=\frac{1}{\sqrt{4\pi D^*_A\, t}}\exp\left[-x^2/4D^*_A\, t\right]$$

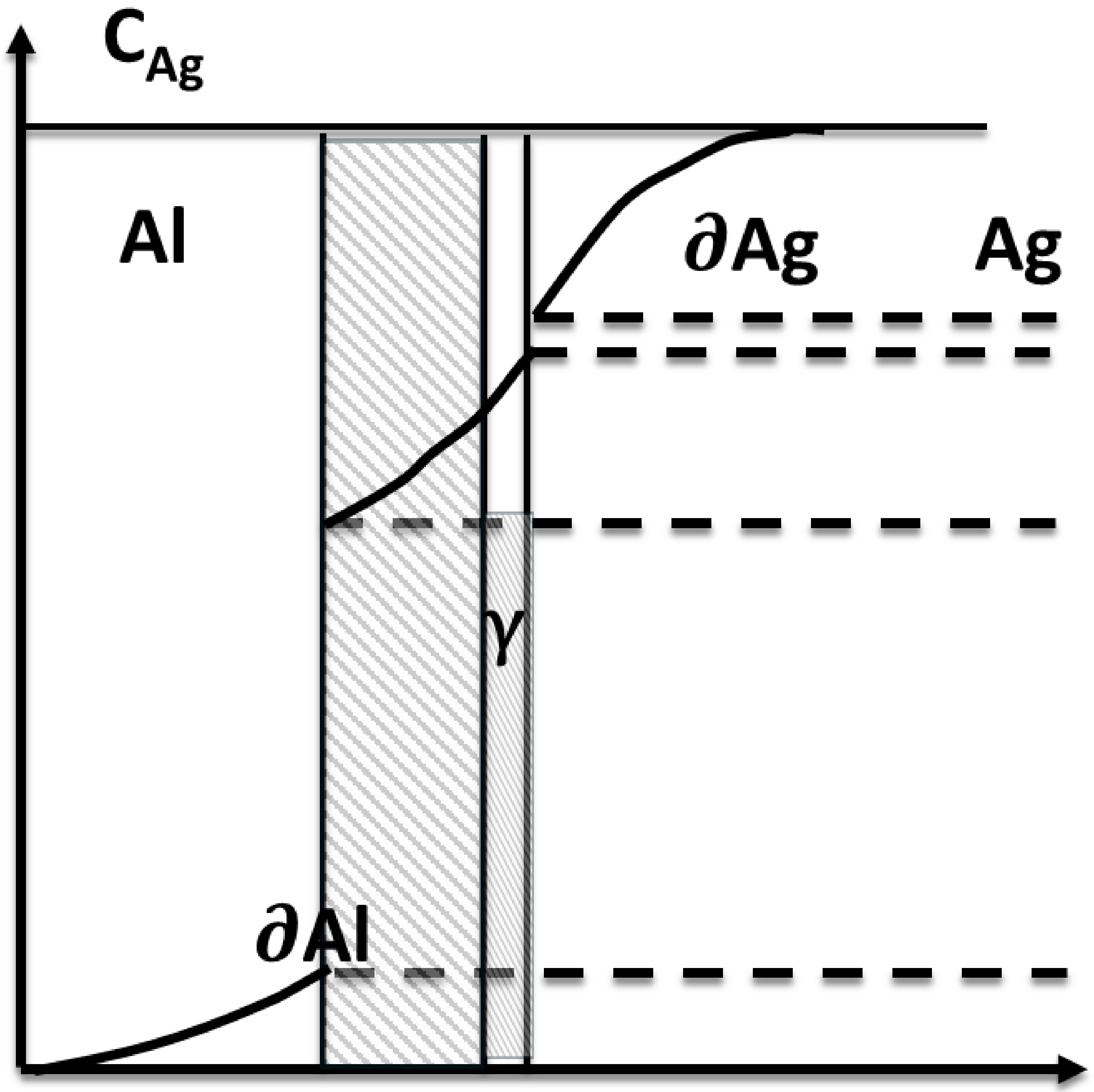
$$C_A(x)=C_0 erf\left(x/2\sqrt{D^*_A t}\right)$$

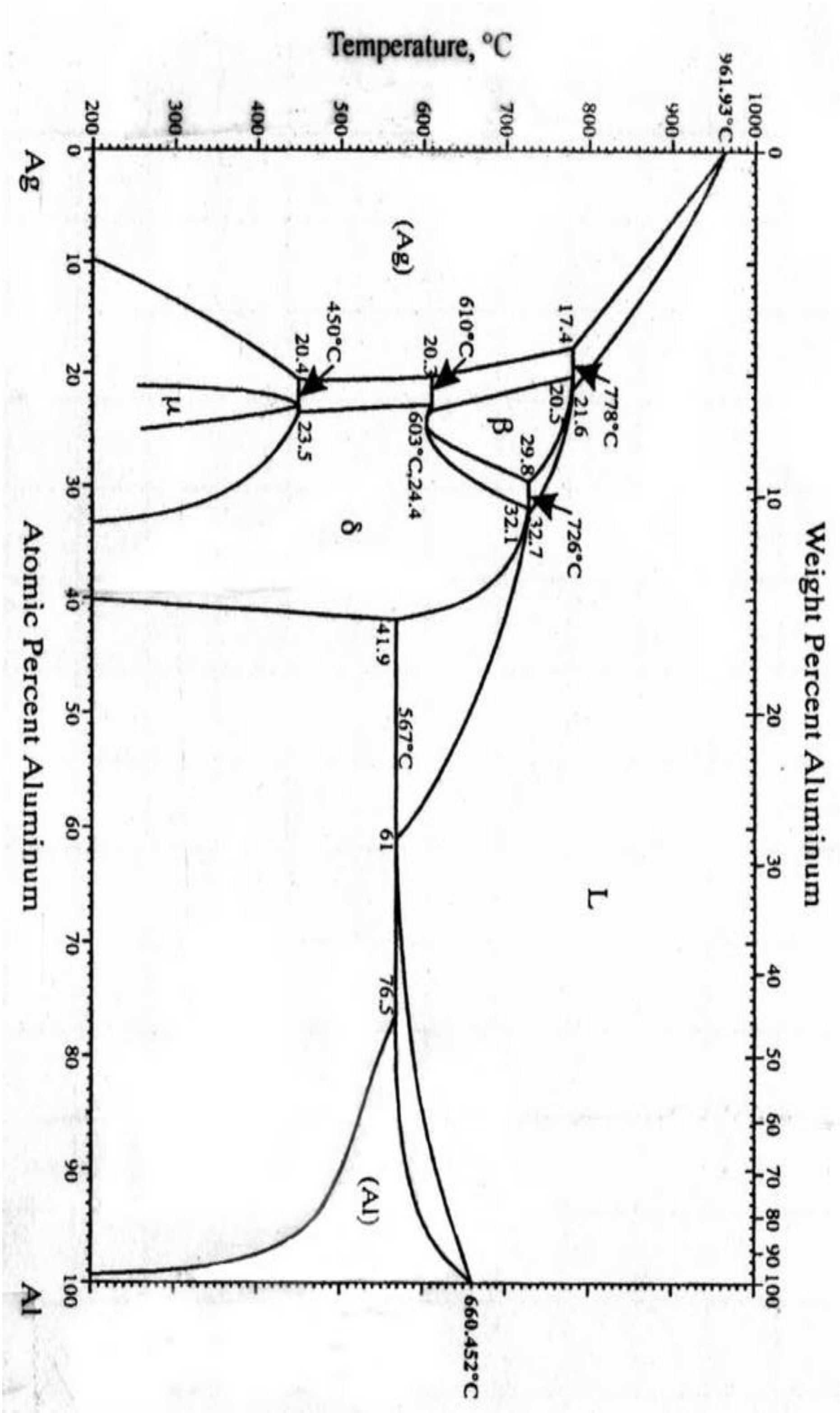
$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2}\,dz$$

In reality, the situation is actually different.....









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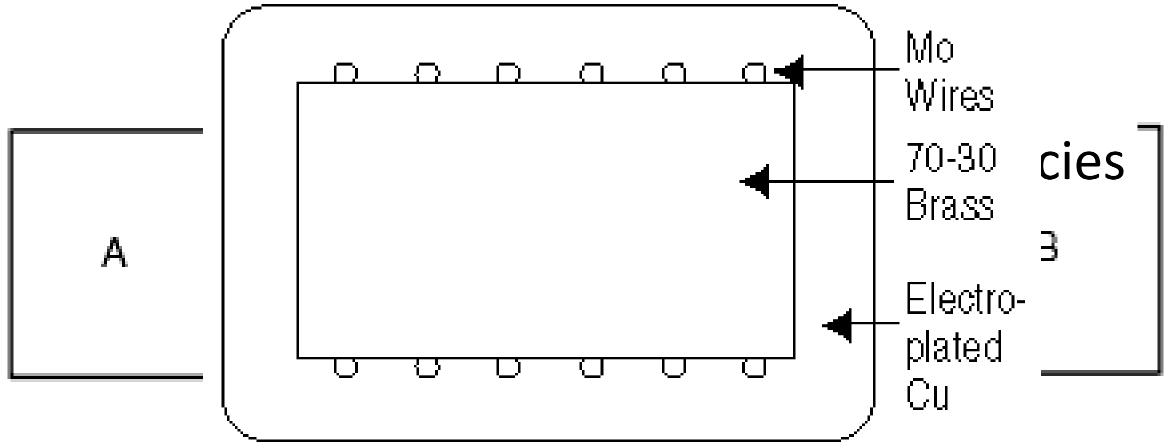
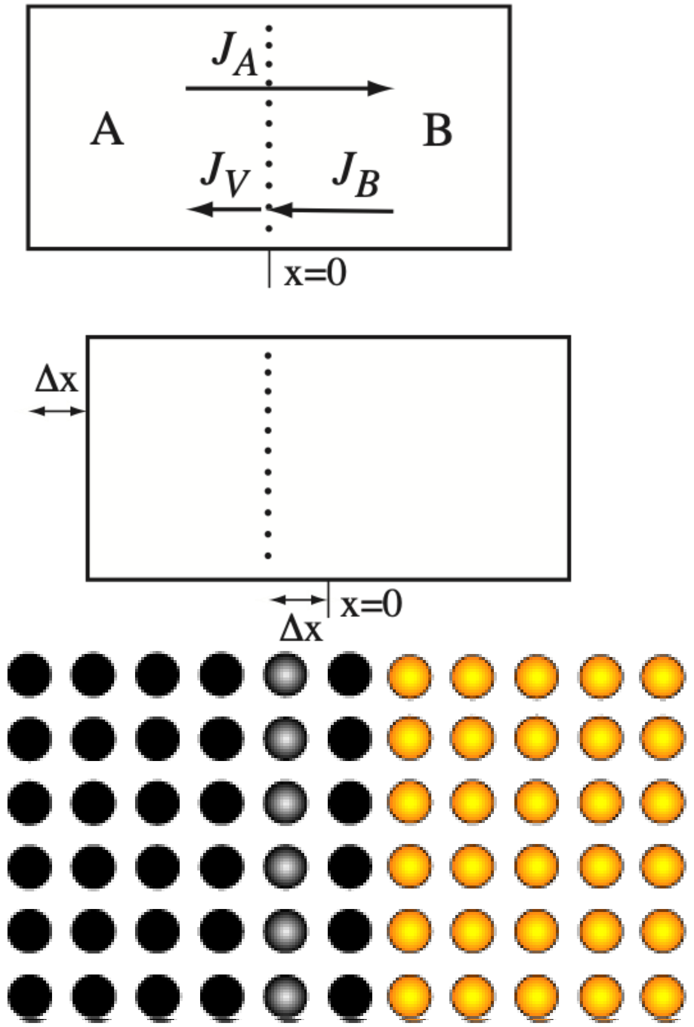
Kirkendall effect

Original experiment (1947)



Da Silva, Mehl configuration (1951)

Explanation: the sample moves depending on the lab reference frame



Kirkendall effect: solving of the problem

$$J_v + J_A + J_B = 0$$ Fixed reference (lab)

$$J_A = -D_A^* \frac{\partial C_A}{\partial x} \qquad J_B = -D_B^* \frac{\partial C_B}{\partial x}$$

Velocity of vacancies

$$v_v = J_v \Omega \qquad v_v = -(J_A + J_B)\Omega$$

Moving reference (sample) $\Rightarrow J_A^0 + J_B^0 = 0 \qquad C_A + C_B = const.$

$$\frac{\partial C_A}{\partial x} = -\frac{\partial C_B}{\partial x} \qquad v_v = (D_A^* - D_B^*)\frac{\partial X_A}{\partial x} \qquad X_A = C_A \cdot \Omega$$

$$J_A^0 = -\tilde{D}\frac{\partial C_A}{\partial x} \qquad J_B^0 = -\tilde{D}\frac{\partial C_B}{\partial x}$$

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Kirkendall effect: solving the problem

$$J_A^0 = J_A + v_v C_A = -D_A^* \frac{\partial C_A}{\partial x} + v_v C_A \qquad J_B^0 = J_B + v_v C_B = -D_B^* \frac{\partial C_A}{\partial x} + v_v C_B$$

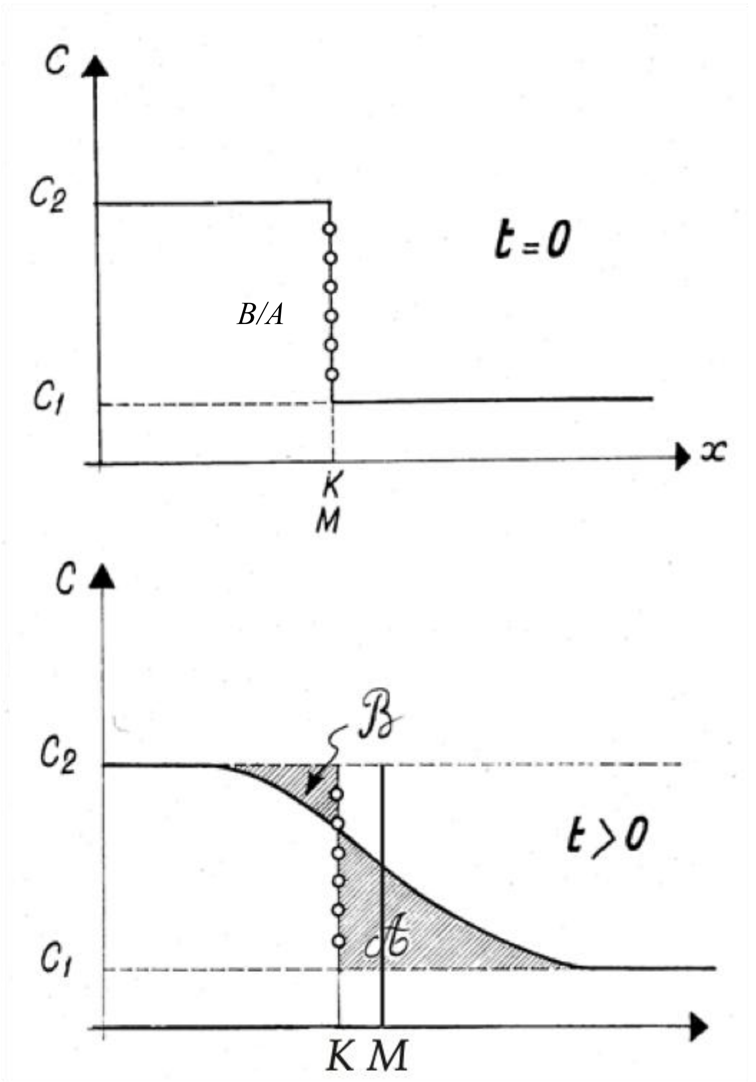
$$J_A^0 == -D_A^* \frac{\partial C_A}{\partial x} + \left(D_A^* - D_B^*\right) \frac{\partial C_A}{\partial x} X_A = -\left(D_A^* - X_A D_A^* + X_A D_B^*\right) \frac{\partial C_A}{\partial x}$$

$$\boxed{\tilde{D} = X_B D_A^* + X_A D_B^*}$$

$$\frac{\partial C_A}{\partial t} = \frac{\partial}{\partial x}\left(\tilde{D} \cdot \frac{\partial C_A}{\partial x}\right)$$

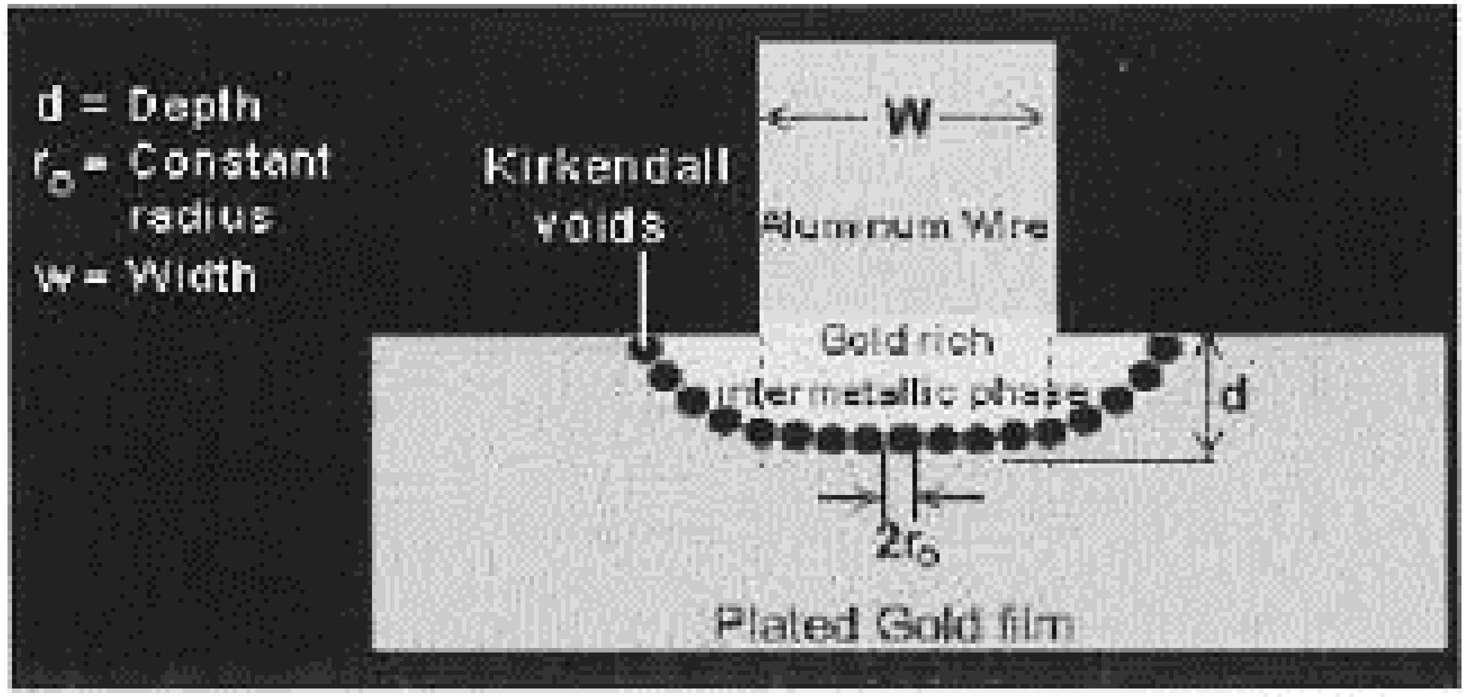
We get $\tilde{D}$ by the B-M method

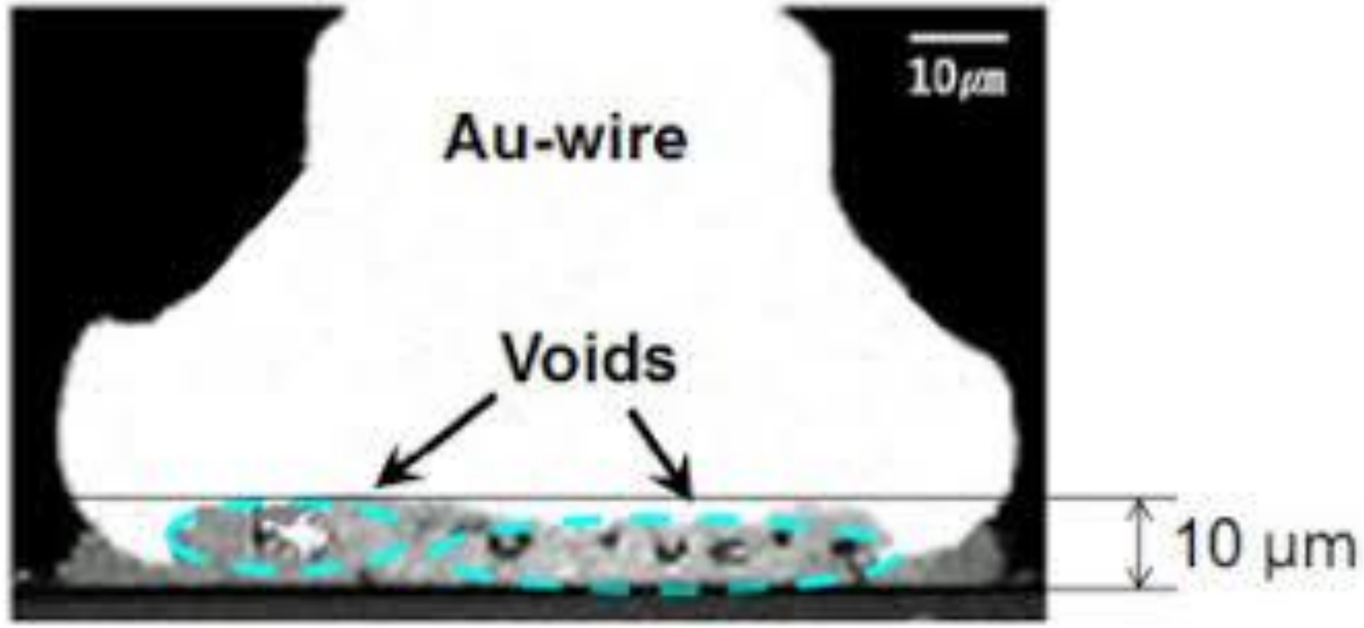
velocity of Kirkendall plane $v_v = (D_A^*) - (D_B^*) \frac{\partial X_A}{\partial x}$

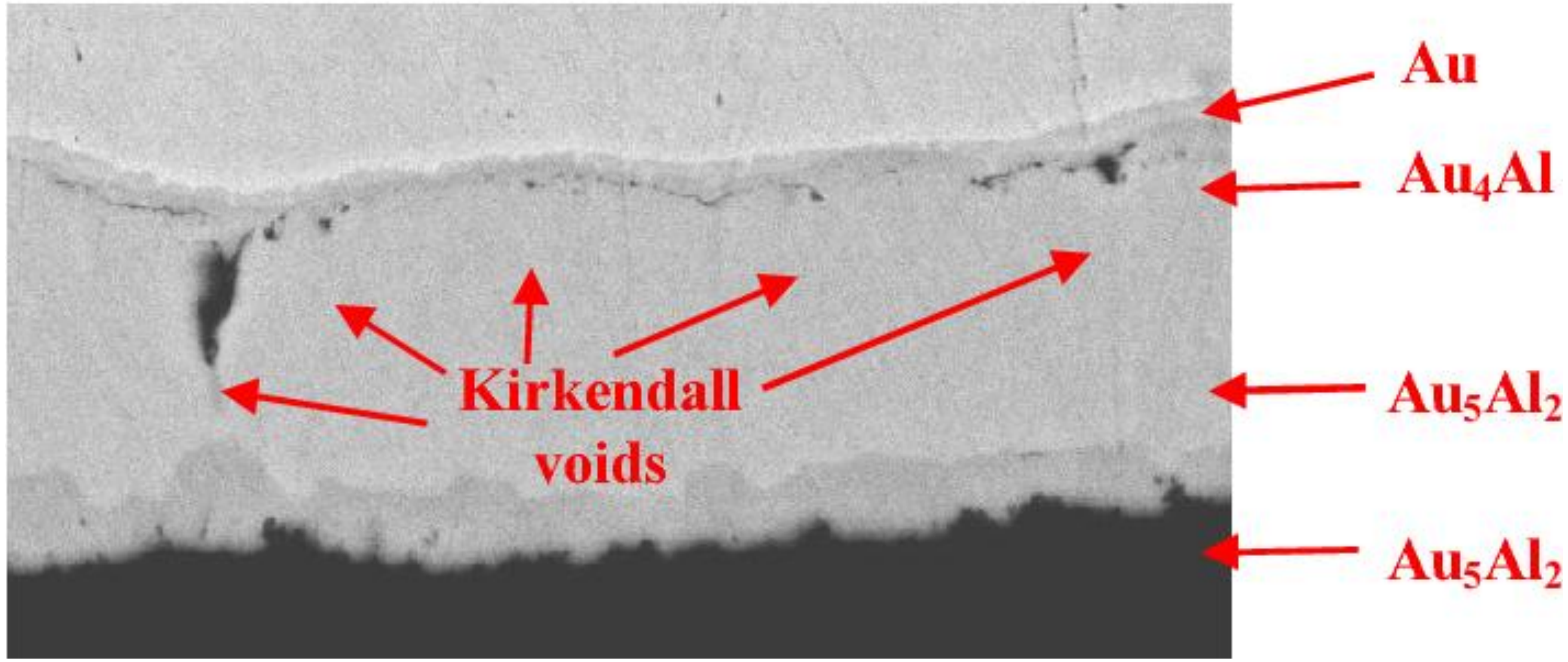


$D^*_B / D^*_A = B/A$

Example of the Kirkendall effect: Wire Bonding







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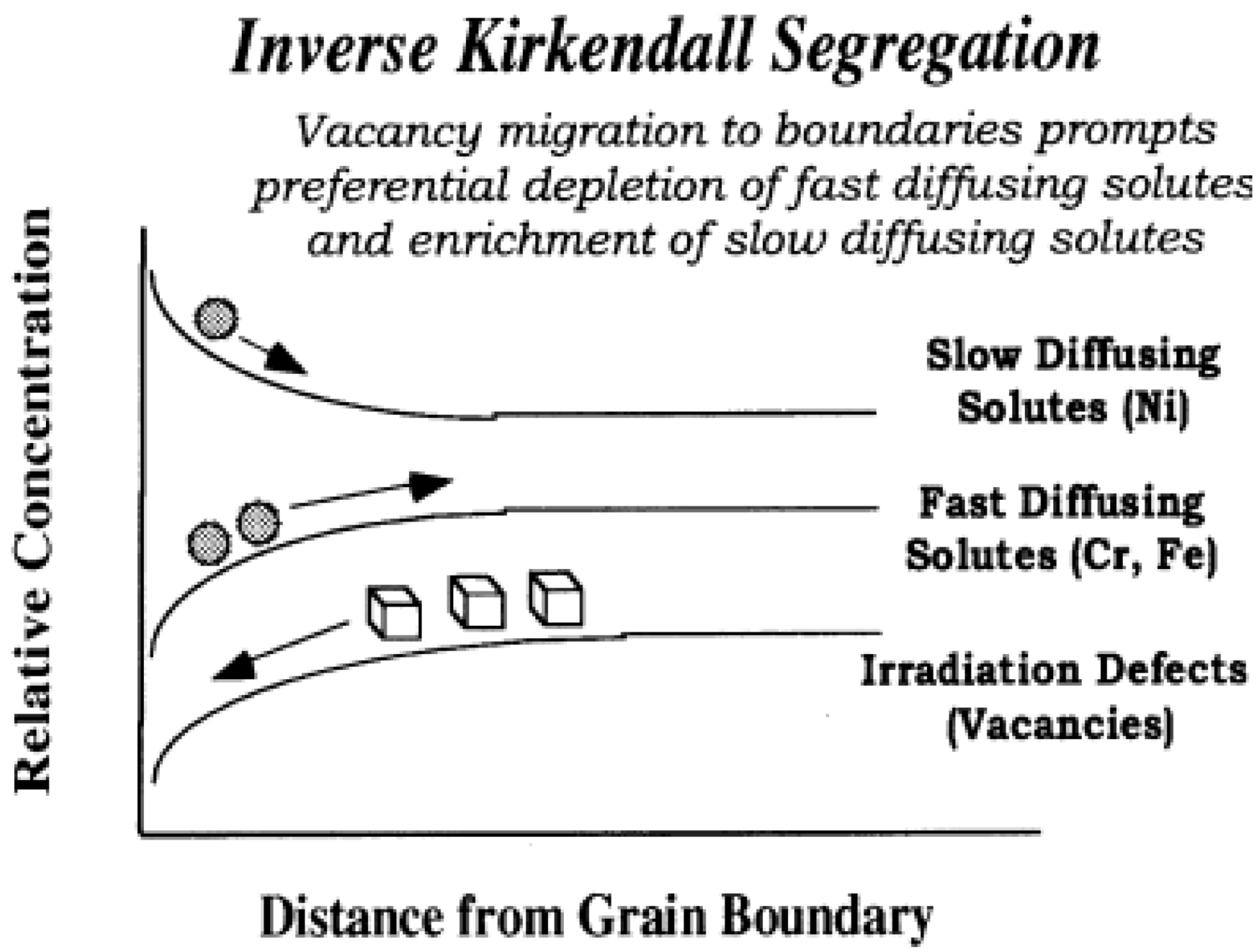
Inverse Kirkendall effect during Neutron Irradiation



Some examples of diffusion coefficients: Fick’s equations

Self diffusion

13–10 *Diffusion in metals*

Table 13.1 SELF-DIFFUSION IN SOLID ELEMENTS—*continued*

Element	*A* $cm^2 s^{-1}$	*Q* $kJ\,mol^{-1}$	*Temp. range* *K*
Group IIB			
Zn	∥c 0.13	91.7	513–691
	⊥c 0.18	96.3	
Cd	∥c 0.118	77.92	420–600
	⊥c 0.183	82.02	
Group IIIA and rare earths			
Y	∥c 0.82	252.5	1 173–1 573
	⊥c 5.2	280.9	
β-La	1.5	188.8	933–1 113
γ-La	0.013	102.6	1 140–1 170
	0.11	125.2	1 151–1 183
γ-Ce	0.55	153.2	801–965
δ-Ce	0.012	90.0	992–1 044
β-Pr	0.087	123.1	1 323–1 473
Er	∥c 3.71	301.66	1 479–1 684
	⊥c 4.51	302.58	
Eu	1.0	144.4	771–1 072
β-Gd	0.01	136.9	1 549–1 581
α-Yb	0.034	146.79	823–983
γ-Yb	0.12	121.0	1 031–1 083
Group IIIB			
Al	1.71	142.4	723–923
	2.25	144.4	573–923
	0.137	123.5	298–581[a]
	0.176	126.4	358–482
Ga	$D = 5.3 \times 10^{-13}$		282.8
	$D = 5.3 \times 10^{-13}$		293.0
	$D = 7.8 \times 10^{-13}$		298.0
	$D = 9.3 \times 10^{-13}$		300.5
	$D = 42 \times 10^{-13}$		302.7
In	∥c 2.7	78.3	317–417
	⊥c 3.7	78.3	
α-Tl	∥c 0.4	95.9	423–498
	⊥c 0.4	94.6	
β-Tl	0.42	80.18	513–573

Diffusion at low concentration

Table 13.2 TRACER IMPURITY DIFFUSION COEFFICIENTS

In Ag

Element	*A* $cm^2 s^{-1}$	*Q* $kJ\,mol^{-1}$	*Temp. range* K	*Method*	*Ref.*
Cu	1.23	193.0	990–1 218	IVa(i), s.c., Cu^{64}, 99.99%	13
	0.029	164.1	699–897	IVa(iii) (SIMS), s.c., 99.99%	258
Au	0.85	202.1	991–1 198	IVa(i), s.c., Au^{195}, 99.99%	18
Zn	0.54	174.6	916–1 197	IVa(i), s.c., Zn^{65}, 99.99%	14
	0.532	174.6	970–1 225	IVa(i), s.c., Zn^{65}, 9.999%	259
Cd	0.44	174.6	866–1 210	IVa(i), s.c., Cd^{115}, 99.99%	10
Hg	0.079	159.5	926–1 1221	IVa(i), s.c., Hg^{203}, 99.99%	13
Al	0.13	159.5	873–1 223	IIb (X-ray), p.c.,	211
Ga	0.42	162.9	873–1 213	IIb (X-ray), p.c., —	260
In	0.41	170.1	886–1 209	IVa(i), s.c., In^{114}, 99.99%	10
	0.36	169.0	553–838	IVa(i), s.c., In^{114}, 99.999%	261
Tl	0.15	158.7	918–1 073	IVa(i), p.c., Tl^{204}, —	15
Ge	0.084	152.8	943–1 123	IVa(i)m p.c., Ge^{71}, —	15
Sn	0.25	165.0	865–1 1210	IVa(i), s.c., Sn^{113}, 99.99%	10
Pb	0.22	159.5	973–1 098	IVa(i), p.c., Pb^{210}, ··	19
As	0.42	149.6	915–1 213	IVa(iii) (EMPA), p.c., 99.98%	107
Sb	0.169	160.4	743–1 215	IVa(i), s.c., Sb^{124}, 99.99%	11
S	1.65	167.5	873–1 173	IVb, s.c., S^{35}, 99.999%	108
Se	0.285	157.4	759–1 109	IVa(r) (ion impl), s.c., Se^{75}, 99.999%	262
Te	0.21	154.7	650–1 169	IVa(i), s.c., Te^{121}, 99.999%	109
Ti	1.33	198	1 051–1 220	IIa(ii) (EMPA), p.c., 99.999%	263
V	2.72	209	1 012–1 218	IVb, p.c., V^{48}, 99.999%	263
Cr	3.29	210	1 023–1 215	IVb, p.c., Cr^{51}, 99.999%	263
	1.07	192.6	976–1 231	IVa(i), s.c., Cr^{51}, 99.9999%	264
Mn	4.29	196	883–1 212	IVb, p.c., Mn^{54}, 99.999%	263
Fe	2.6	205.2	1 073–1 205	IVa(i), s.c., Fe^{59}, 99.999%	106
Ru	180	275.5	1 066–1 219	IVa(i), s.c., $Ru^{103/106}$, 99.99%	12
Co	1.9	204.1	973–1 214	IVa(i), s.c., Co^{60}, 99.999%	106

Some examples of diffusion coefficients: chemical diffusion

Chemical diffusion

Table 13.4 CHEMICAL DIFFUSION COEFFICIENT MEASUREMENTS—*continued*

Element 1 At.%	*Element2* At.%	*A* $cm^2 s^{-1}$	*Q* $kJ mol^{-1}$	*D* $cm^2 s^{-1}$	*Temp. range* K
Mo	Pd				
	61	5.5×10^{-5}	188.4	($D_{Pd} \approx$ [a]	
	66	4.0×10^{-5}	165.4	$10–20\times D_{Mo}$)	
	71	5.0×10^{-5}	177.9	—	
	75	2.4×10^{-4}	200.5	—	
	80	1.6×10^{-3}	218.6	—	1 273–1 873
	85	1.6×10^{-2}	253.3	—	
	90	9.0×10^{-1}	293.1	—	
	95	1.4×10^{-1}	282.6	—	

(*a*) At the original composition in pure Mo/pure Nb couples.

Element 1 At.%	*Element2* At.%	*A* $cm^2 s^{-1}$	*Q* $kJ mol^{-1}$	*D* $cm^2 s^{-1}$	*Temp. range* K
Mo	Ta				
	'Ta rich'	4.68×10^{-3}	251.2	—	2 175–2 573
	'Mo rich'	4.16×10^{-3}	234.5	—	
Mo	Ti				
0		$\sim2\times10^{-2}$	196.8	—	
10		$\sim2\times10^{-2}$	209.3	D_{Ti}/D_{Mo}	
20	(β)	$\sim1\times10^{-2}$	217.7	~3 at 1 600° ~13 at 820°	1 483–1 873
30		$\sim10^{-2}$	263.8	—	
40 [a]		$\sim10^{-2}$	255.4	—	
0–10	(β)	1.3×10^{-4}	138.6	—	1 173–1 573
Sol. soln range	(α)	3.5×10^{-8}	118.9	—	873–1 073

Diffusion in binary alloys

Table 13.3 DIFFUSION IN HOMOGENEOUS ALLOYS—*continued*

Element 1 (*purity*) At.%	*Element 2* (*purity*) At.%	*A*‡ $cm^2 s^{-1}$	*Q*‡ $kJ mol^{-1}$	*A*‡ $cm^2 s^{-1}$	*Q*‡ $kJ mol^{-1}$	*Temp. range* K	*Ref.*
Co	Cr						
(--)	(—)		IVb	p.c.			
	4	0.67	275.5	—	—	1 373–1 623	33
	7	56.3	332.0	—	--		
Co	Cr Ni						
( )	(—) (--)		IVb	p.c.			
	9 26	6.3	301.9		--	1 373–1 623	33
	18 26	0.4	268.8	--	—		
Co	Cu Si						
(99.99)	(99.99) ( )			IVa(i), p.c., Co^{58}			
0–2 68	0.54		$B_1 \approx -9$			1 325	158
Co	Fe						
( )	( )		IVa(i)	p.c.			
		1.33	290.6	1.26	286.8	1 285–1 437(γ)	
50	50	2.0	251.2	0.25	230.3	1068–1218(α)	34
			556.8		556.8	928–995(CsCl)	
( )	( )		IVb	p.c.			
—	21	0.54	272.1	—	—	1 373–1 573	33
( )	( )		IVa(iii)	p.c.			
			Diffusion of Ni				
.	0		$D_{Ni} = 13.10^{-12}$				
	30.2		= 11.5				
--	68.5		≈ 8–8.8			1 409	82
---	88.3		~5				
—	100		≈ 6				
( )	( )		IVb	p.c.	(F ≈ Ferromagnetic)		
0	(bcc)	1.83	234.0	—		1 073–1 172	
	(fcc)	0.77)	265.0	. .	—	1 223–1 633	
3	(bcc)	9.17	266.3	—	—	903–1 023(F)	
6.8	(bcc)	0.469	187.1	—	..	903–1 073(F)	
		5.72×10^{-5}	146.5	---	—	1 153–1 193	
	(fcc)	0.109	326.2	—	- -	1 283–1 583	
28.6	(bcc)	1.25×10^{-3}	198.0	.	—	903–1 083(F)	
	(fcc)	3.36×10^{-2}	266.3	—	—	1 333–1 583	107
49.6	(bcc)	6.59×10^{-2}	247.0	..	—	1 023–1 123(F)	
	(fcc)	0.154	349.6	...	—	1 333–1 583	
67.2	(bcc)	6.04×10^{-3}	190.9	—	---	903–1 153(F)	
	(fcc)	3.15×10^{-2}	265.0	—	.	1 333–1 583	
89.6	(fcc)	6.44×10^{-2}	251.2	..	—	1 073–1 283(F)	
		1.61×10^{-2}	234.0	—	---	1 333–1 153	
100	(fcc)	0.50	273.8	..	—	1 045–1 321(F)	
		0.17	260.4	—	—	1 465–1 570	
(99 8)	(99.9)	IVb	p.c.	Co^{60}			
0		0.029	247.4	--	—		
8		20.54	321.5	—	—		
10		15.65	369.0	—	—	1 233–1 493	131
15		1.98	289.7	—	—		
20		0.31	261.7	—	—		
Co	Fe						
( )	( )	IVb, s.c., Fe^{59}					
	6	—	—	0.58	273.3	1081–T_c	
.	6	—	—	0.15	261.7	T_c–1 573	176
.	10	—	—	0.68	279.3	1081–T_c	
--	10	—	—	0.18	263.3	T_c–1 573	

Some examples of diffusion coefficients: interdiffusion

Co	Fe			
( )	( )		IVa(i)	p.c.
		1.33	290.6	1.26
50	50	2.0	251.2	0.25
			556.8	
( )	( )		IVb	p.c.
—	21	0.54	272.1	—
( )	( )		IVa(iii)	p.c.
			Diffusion of Ni	
.	0		$D_{Ni}=13.10^{-12}$	
	30.2		=11.5	
—	68.5		≈8–8.8	
—	88.3		~5	
—	100		~6	
( )	( )		IVb	p.c.
0	(bcc)	1.83	234.0	—
	(fcc)	0.77)	265.0	
3	(bcc)	9.17	266.3	—
6.8	(bcc)	0.469	187.1	—
		5.72×10^{-5}	146.5	—
	(fcc)	0.109	326.2	—
28.6	(bcc)	1.25×10^{-3}	198.0	
	(fcc)	3.36×10^{-2}	266.3	—
49.6	(bcc)	6.59×10^{-2}	247.0	
	(fcc)	0.154	349.6	—
67.2	(bcc)	6.04×10^{-3}	190.9	—
	(fcc)	3.15×10^{-2}	265.0	—
89.6	(fcc)	6.44×10^{-2}	251.2	—
		1.61×10^{-2}	234.0	—
100	(fcc)	0.50	273.8	—
		0.17	260.4	—
(99.8)	(99.9)	IVb	p.c.	Co^{60}
0		0.029	247.4	—
8		20.54	321.5	—
10		15.65	369.0	—
15		1.98	289.7	—
20		0.31	261.7	—
Co	Fe			
( )	( )	IVb, s.c., Fe^{59}		
	6	—	—	0.58
.	6	—	—	0.15
.	10	—	—	0.68
—	10	—	—	0.18

Interdiffusion coefficient

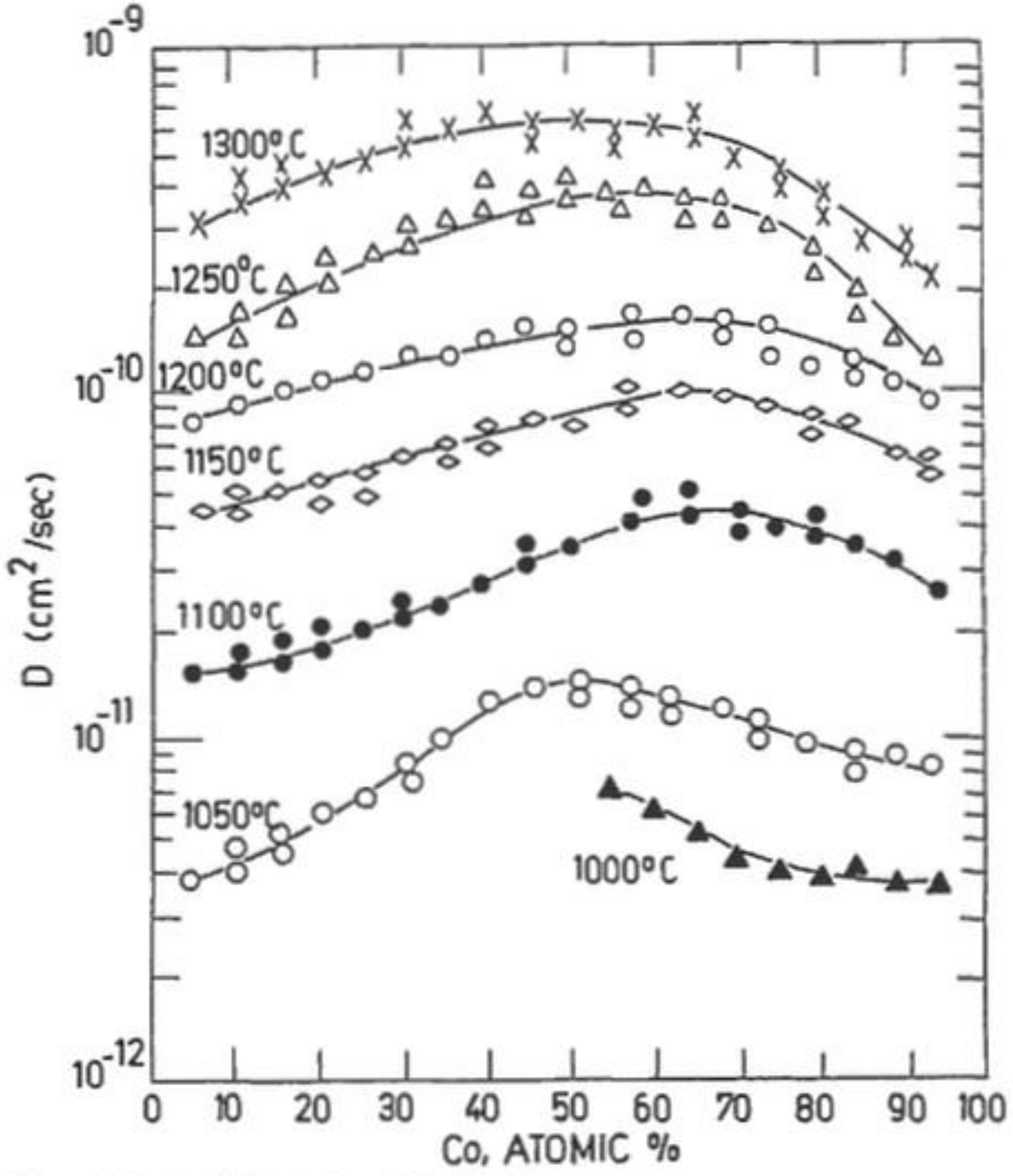


Figure 13.6b *Interdiffusion in f.c.c. FeCo alloys*[318]